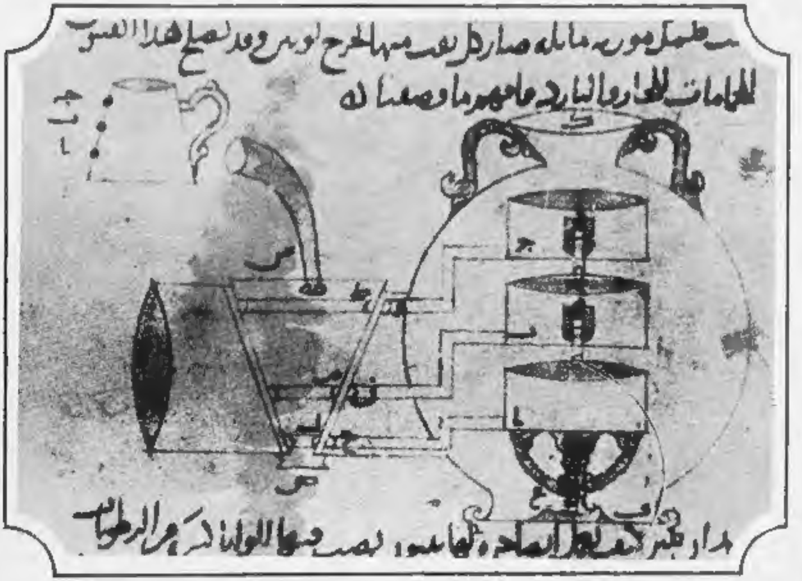
مجلة تاريخ العلوم العربية



معهد التراث العلمي العربي

جامعة حلب ـ سورية

مجلة تاريخ العلوم العربية

المجلد الثالث | العدد الاول | ربيع ١٩٧٩

محتويات العدد

القسم العربي

الابحـاث :

ملخصات الابحاث المنشورة في القسم الاجنبي

مقالة قصيرة واعلانات :

مراجعات الكتب

مجلة تاريخ العلوم العربية

تصدر مجلة تاريخ العلوم العربية عن معهد التراث العلمي العربي مرتين كل عام (في فصلي الربيع والخريف) . يرجى ارسال نسختين من كل بحث أو مقال الى :
معهد التراث العلمي العربي ـ جامعة حلب .

توجه كافة المراسلات الخاصة بالاشتراكات والاعلانات والأمور الادارية الى العنوان نفسه . يرسل المبلغ المطلوب من خارج سورية بالدولارات الاميركية بموجب شيكات باسم
الجمعية السورية لتاريخ العلوم

قيمة الاشتراك السنوي :

المجلد الاول أو الثاني (١٩٧٧ . ١٩٧٨)
بالبريد العادي المسجل : ٢٥ ليرة سورية أو ٦ دولارات أميركية
بالبريد الجوي المسجل : ٤٢ ليرة سورية أو ١٠ دولارات أميركية

المجلد الثالث (١٩٧٩)
بالبريد العادي المسجل : كافة البلدان ١٠ دولارات أميركية
بالبريد الجوي المسجل : البلاد العربية والاوروبية ١٢ دولارا أميركيا
آسيا وأفريقيا ١٥ دولارا أميركيا
الولايات المتحدة ، كندا واستراليا ١٧ دولارا أميركيا

مطبعة جامعة حلب

رسالة أبي جعفر الخازن في المثلثات القائمة الزوايا المنطقة الأضلاع

نشر وتحقيق عادل أنبوبا

رسالة أبي جعفر [الخازن] في المثلثات العددية التي ننشرها اليوم، قد نجا منها نسخة فريدة جاءت ضمن مجموعة ثمينة تحتفظ بها المكتبة الوطنية بباريس تحت رقم ٢٤٥٧ . وتضم المجموعة ٥١ مقالا او قطعة، زعم قبكه ان اكثرها بخط الرياضي المعروف ابي سعيد السجزي؛ ومن منسوخات السجزي ما يحمل تاريخ كَتْبه ٣٥٨ ، ٣٥٩ ه او موضعه مدينة شيراز . ولم تُذكر هذه الرسالة في المؤلفات القديمة المحفوظة إلا انه يحتمل ان تكون هي احدى الرسائل التي دلّ عليها صاحب الفهرست ابن النديم اذ قال مجملا : كتاب المسائل العددية لابي جعفر الخازن[1] . وابو جعفر الخازن هو ابو جعفر محمد بن الحسين الخراساني الصاغاني الخازن ، رياضي وفلكي ازهر في النصف الاول من القرن الرابع الهجري وتوفي بعيد ٣٥٠ ه — وقد اشرنا الى جُمل من حياته في مقال لنا سابق في هذه المجلة[2] . ونُبيّن في الجزء الفرنسي من هذا المقال موضع الرسالة من تطور العلوم عند العرب ، وترجمة لمعانيها مع بعض الملاحظات والايضاحات .

المخطوط :

تحوي الصفحة الواحدة من المخطوط ٢٢ سطرا او ما يقرب وقد كتب المخطوط بخط عادي واضح قليل الاخطاء . الكثير من الحروف غير منقطة سيما حروف المضارعة وقد

١ — الفهرست طبعة القاهرة دون تاريخ ص ٤٠٧ . وذكر المسائل العددية ابن القفطي في اخبار الحكماء القاهرة ١٣٢٦ ، ص ٢٥٩ .

٢ — الجبر عند العرب في القرنين التاسع والعاشر للميلاد ، باللغة الفرنسية ، المجلد الثاني (١٩٧٨) ، ص ٦٦—١٠٠ .

اثبتنا النقاط دون الاشارة الى ذلك إلا عرَضا . نضع بعد الكلمة المصححة عددا وبعد مثيله في الحاشية السفلى اللفظة كما وردت في المخطوط واذا شمل التصحيح بضع كلمات وضعناها بين علامتين « » . والعدد داخل مكوفين مثل [١] يشير الى لفظة حذفناها من النص على انها زائدة وذكرناها في الحاشية بعد العدد وتدل العلامة] [ان بين المكوفين ما نرجح انه من زيادة خاطئة للناسخ . وكتبنا ب̄ ل̄ ج̄ د̄ ... بدلا من كل ب̄ جد .. في الدلالة على الخطوط .

ثم انا قطّعنا النص فِقرا تسهيلا للمطالعة وتمييزاً لمعانيه .

ويسرنا ان نتوجه هنا بخالص الشكر الى المكتبة الوطنية بباريس والآنسة الكريمة M.-R. Séguy حافظة المخطوطات الشرقية التي تفضلت واذنت لنا بنشر المخطوط .

كنا قد أودعنا هذا المقال ادارة المجلة في أيار ١٩٧٨ أو قبله ، ثم تأخر نشره لعوارض عرضت. وقد نشر في هذه الأثناء الدكتور أحمد سعيدان رسالة أبي جعفر الخازن هــذه مع شروح وتعليقات وملخص انكليزي (مجلة الدراسات ، كانــون الاول ١٩٧٨ ، الجامعــة الاردنية) . وقد أشرنا الى بعض قراءات مفيدة للدكتور سعيدان تحت حرف س .

رسالة ابي جعفر [الخازن] في المثلثات القائمة الزوايا
المنطقة الاضلاع ، باريس مخطوط ٢٤٥٧ ، ص ٢٠٤ أ ـــ ٢١٥ أ .

٢٠٤ أ بسم الله الرحمن الرحيم

رسالة الشيخ ابي جعفر محمد بن الحسين ايده الله* الى عبد الله بن علي الحاسب في البرهان على انه لا يمكن ان يكون ضلعتا عددين مربعين يكون مجموعهما مربعاً فردين بل يكونان زوجين او احدهما زوج والاخر فرد تتلوا رسالته اليه في انشاء المثلثات القائمة الزوايا المنطقة الاضلاع .

* – كذا والصحيح ان يقع الدعاء " ايده الله" بعد اسم المرسل اليه : عبد الله بن علي الحاسب ايده الله . والمقالة تعنى بالاعداد الصحيحة إلا في مواقع قليلة يشير اليها النص .

1 1 كنت قد بيّنْتُ فيما كتبتُ به اليك اخي ايدك الله في نشوء[1] المثلثات القائمة الزوايا المنطقة الاضلاع انه لا يمكن ان يكون ضلعا عددين مربعين يكون مجموعهما مربعاً فردين بل يكونان زوجين او يكون احدهما زوجا والآخر فردا ولم ابرهن على ذلك بشكل خطوطي[b] فرأيت ان ابيّنه به ليقع تحت الحسّ واذكر معه ما يتصل معناه بما كتبت ويزيده بيانا[2] ويفيد[3] الناظر فيه يقينا[4] وهذا ابتداؤه نريد ان نبيّن كيف تنشأ[5] الاعداد المربعة التي يكون مجموع كل عددين منها مربعا فنقدم لذلك ثلث مقدمات :

2 2 احداها[6] : انه لا يمكن ان يوجد عددان مربعان فردان يكون مجموعهما مربعا فان
امكن فليكن عددا ا ب مربعين فردين وليكن مجموعهما وهو ج مربعا فيكون زوجا مما
بيّن في **الشكل الثاني والعشرين من المقالة التاسعة**[7] **من كتاب الاصول** ونجعل د ه ضلع
ا و ز ضلع ب و ط ح ضلع ج وتفصل من ط ح مثل ز وهو ك ح فلان ط ح زوج و ك ح
فرد يبقى ط ك واحدا او عددا فردا ونحسبه اولاً واحداً[8] ونزيد في ك ح مثله وهو ل ح
فيكون ضرب ط ل في ط ك مثل مربع د ه ولكن ضرب ط ل في ط ك مثل مربع ط ك
مع ضرب ك ل في ط ك فمربع ط ك مع ضرب ك ل[9] في ط ك مثل مربع د ه ونفصل من
د ه مثل ط ك وهو م ه ونزيد فيه مثل م ه وهو ه ن فيكون ضرب د ن في د م مع مربع
م ه مثل مربع د ه ونسقط مربع م ه الذي هو مثل مربع ط ك فيبقى ضرب د ن في د م
٢ب ٢ب مثل ضرب ك ل في ط ك فلان ط ك واحد يكون ضرب ك ل | في ط ك هو ك ل وضرب
د ن في د م مثل ك ل وكل واحد من د ن د م زوج لان د ه فرد وقد نقص منه واحد
وزيد عليه واحد فيكون د ن الزوج في نصف د م الزوج مثل نصف ك ل وهو فرد لانه
مثل ز الفرد فليس ك ط بواحد .

b – اي باستعمال الخطوط الدالة على الاعداد كما في مقالات اقليدس ٧ ، ٨ ، ٩ مثلا .

١ – نشو

٢ – بانا

٣ – يفيده

٤ – تفننا

٥ – تنشأ

٦ – احداهما ٧ – في النص الثامنة والتصحيح جاء في الهامش

٨ – فردا ٩ – ط ل

الىعدد مربع ومضروب احدهما في الآخر اربع مرات ومضروبه فيه مرة واحدة عددان مربعان ومثل ثمنية واثنين فانهما بهذه الصفة . فلإن **العدد المركب والفضلة** بهــذه الحالة **وهما ايضا** اقل هذه الاعداد من قبل انا جعلنا العددين المطلوبين اللذين انزلناهما موجودين **اقل عددين مربعين** وجب ان يكون كل واحد من **العدد المركب والفضلة** مربعا ويكون الفضلة واحدا[٦] اذ هو اقل المربعات والعدد المركب اربعة اذ هي اقل الاعداد المربعات وان يكون العدد الفرد منه ثلثة وهو ضلع احد المربعين المطلوبين والمربع تسعة ويكون المربع الآخر مضروب الاربعة في الواحد اربع مرات الــذي هو مضروب الاربعة في نفسها وهو ستة عشر وضلع مجموعهما[٧] خمسة وهي مجموع ضــلع المربع الفرد وضعف [٨] الفضلة . فقــد ظهر من ذلك ان فضل ما بين المربعين اللذين هما اربعة وواحد وهو ثلثة ضلــع احــد المربعين المطلوبين وان مضروب ضعف ضلع المربع الاقل وهو اثنان في ضلع المربع الاكثر الذي هو اثنان وهو اربعة ضلع المربع الآخر من المربعين المطلوبين وان مجموع المربعين الذي هو خمسة ضلع مجموع المربعين المطلوبين .

6 وهذا الطريق مطرد في وجود سائر الاعداد المربعة التي يكون مجموع كل اثنين منها مربعا ؛ فانا اذا اخذنا فضل ما بين التسعة والواحد المربعين وهو ثمنية واخذنا مضروب
١٢٠٦ ضعف ضلع الواحد في ضلع التسعة وهو ستة وضربنا كل واحد في مثله اجتمع اربعة وستون وستة وثلثون وكان مجموعهما مابة وضلعه عشرة مثل مجموع المربعين الاولين g إلا ان ضلع كل واحد من المربعين ومن مجموعهما ضعف كل واحد من المربعين الاولين ومن مجموعهما فاضلاعها مشارك بعضها لبعض h . وكذلك كل عددين مربعين يكون نسبــة ضلع احدهما الى ضلع الآخر كنسبة اربعة الى ثلثة فانهما يكونان مركبين من هذين العددين ويعد ضلع مجموعهما الخمسة وذلك بيّن . وكذلك[٩] لا يعسر وجود الاعداد المربعة التي اذا زدنا على كل واحد منها واحدا عد مجموعهما الخمسة i .

٦ — واحد
٧ — مجموعها   ٨ — ضلع
g — يعني ان ١٠ مجموع المربعين الا ساسين ١ و ٩ اللذين نشأت عنهما الاعداد المربعات الثلاث
h — اي ان ٦ ٨ ١٠ ضعف ٣ ٤ ٥ التي نشأت عن المربعين الاساسيين ١ و ٤
٩ — ولذلك
i — يعني ان المربع الا ساسي ١ لما اخذ مع ٤ او مع ٩ نشأ عن ذلك ١ + ٤ = ٥ ١ + ٩ = ١٠ والخمسة تعد ٥ و ١٠ ولا يصعب ان نجد مربعات مثل ٤٩ ، ١٦٩ ، ٦٤ ، ١٤٤ ، اذا اضيفت الى ١ نشأ عن ذلك اعداد تعدها ٥ وهي مربعات كل عدد ينتهي برقم ٢ ، ٣ ، ٨ ، ٧

7 فينبغي ان نطلب غير ذلك وهو ان نطلب العددين اللذين بعد تسعة وستة عشر j واذا[١] كان الواحد والتسعة عند مجموعَهما الخمسة فنأخذ العددين المربعين اللذين يليان الواحد والاربعة وهما اربعة وتسعة فيكون فضل مابينهما وهو خمسة ضلع احد المربعين المطلوبين ومضروب ضعف ضلع الاربعة وهو اربعة في ضلع التسعة وهو اثنا عشر ضلع المربع الآخر والخمسة والاثنا عشر **اصل الاعداد** التي كل اثنين منها على نسبتهما فاحد المربعين خمسة وعشرون والآخر مائة واربعة واربعون وضلع مجموعهما وهو مائة وتسعة وستون ثلثة عشر وهي مجموع المربعين المأخوذين . ونطلب العددين التاليين للاربعة والتسعة وهما واحد وستة عشر فيكون ضلع المربع الاقل ثمنية وضلع الاكثر خمسة عشر وضلع مجموعهما وهو مايتان وتسعة وثمنون سبعة عشر فهي مجموع المربعين .

8 وعلى ذلك تنشأ اضلاع هذه المربعات بان يؤخذ كل عددين مربعين يكونان **اقل عددين على نسبتهما واقل عددين على نسبة عددين هما متباينان** مثل الواحد والاربعة فانهما متباينان لان الواحد يعد كل عدد وكذلك اربعة وتسعة وواحد وستة عشر فيعمل بهما ما وصفنا من العمل فينشأ منها الاعداد المربعة التي يكون مجموع كل عددين منها مربعا من
٢ب غير ان يكون بين عددين وعددين | منها عددان على صورة العددين اللذين قبلهما k فانه لا يوجد مثل[٢] ستة عشر وتسعة عددان بهذه الصورة غيرهما ولا غير[٣] مائة واربعة واربعين وخمسة وعشرين وغير اربعة وستين ومائتين وخمسة وعشرين .

9 فان أخذ اربعة ومائة واحد وعشرين **وهما متباينان** ومجموعهما تعده الخمسة فانه ينشأ منهما عددان مربعان مجموعهما مربع لا يعد ضلعيهما ضلعا الستة عشر والتسعة **بالسوية** وهما اربعة واربعون ومائة وسبعة عشر وعلة ذلك ان مائة وخمسة وعشرين مركبة من الخمسة والخمسة والعشرين وكل واحد منهما ينقسم بعددين مربعين وكل عدد هذه صورته فانه ينقسم بعددين مربعين مرتين كما نبيّن ذلك فيما بعد فقد انقسم مائة وخمسة وعشرون

١ – اذ

j – بعد ان استعمل $١^{٢}$ و$٢^{٢}$ و $١^{٢}$ و$٣^{٢}$ و$٢^{٢}$ و $٤^{٢}$ لتركيب المربعات منها فانه يستعمل $٢^{٢}$ و $٣^{٢}$. وليس النص واضحا على كل حال

k – فحوى الكلام انه لن نحصل على عددين متناسبين لعددين سابقين فلن نحصل على عددين متناسبين لـ ١٦ ، ٩ ؛ او ٦٤ و ٢٢٥ الخ .

٢ – بين ٣ – بعد

مرة اولى بخمسة وعشرين وبماية ومرة اخرى باربعة وبماية وأحد وعشرين فكل عدد يكون بهذه الصورة فسبيله سبيل الخمسة فان ضلعي مربعي[4] قسميها وهما اربعة وثلثة هما اصلان للاعداد المركبة من الخمسة مثل الماية فانها تنقسم بستة وثلثين واربعة وستين وضلع ستة وثلثين مركب من الثلثة وضلع اربع وستين مركب من الاربعة والستة والثمنية يعدهما الثلثة والاربعة **بعدد واحد** وهو الاثنان فينبغي ان نعلم ذلك من خواص هذه الاعداد .

10 فان كان العددان المربعان زوجين نقصنا من ضلع مجموعهما ضلع اقلهما فيكون الباقي زوجا ونضيف نصفه[1] وهو **الفضلة** الى ضلع المربع الاقل فيكون ضرب مجموعهما في **الفضلة** مربعا اذ كان ضربه في اربعة اضعافها كما بيّنا مربعا وضلعه نصف ضلع المربع الاكثر[5] من المربعين الاولين . فقد ظهر مما قلنا ان كل عدد مربع ينقسم بعددين مربعين فان ضلعه ينقسم بعددين مربعين مشتركين او متباينين او ينقسم بسطحين متشابهين .

11 وقد يمكن ان نجد عددين مربعين مجموعهما مربع وثلثة اعداد مربعة مجموعها

١٢٠٧ أ مربع | وكذلك اربعة وخمسة والى غير نهاية. فوجود عددين مربعين مجموعهما مربع ان نأخذ عددين مربعين ونضرب احدهما في الآخر فيخرج احد المربعين ¤ ونأخذ مربع نصف فضل ما بينهما ¤[6] فيخرج المربع الآخر ويكون مجموعهما مربعاً ضلعه نصف الاكثر مع الاقل¤¤ . مثال ذلك ان نفرض اج[7] بج عددين مربعين ونجعل د مضروب اج في بج وننصّف اب على ه فاقول ان مجموع [ ][8] د ومربع هب ضلعه هج . برهان ذلك ان ضرب اج[7] في بج وهو د مربع **كما تبيّن في المقالة التاسعة من كتاب الاصول**¤ ؛ وضرب اج[7] في بج مثل ضرب اب في بج ومربع بج فضرب اب في بج الذي هو مثل ضرب هب في بج مرتين مع مربع بج مثل د ولكن ضرب هب في بج مرتين ومربعي هب بج مثل مربع هج فمربع هج مثل مجموع د ومربع هب فقد وجدنا عددين مربعين مجموعهما مربع ضلعه هج فان كان اج بج مربعين زوجين

٤ – مربعين

١ – نصفه اي نصف الباقي

٥ – الا كر

٦ – ونضرب نصف اكثرهما في مثله

٧ – اه

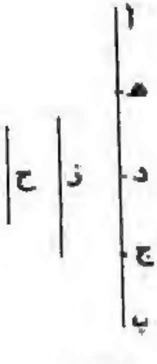
¤¤ – اي نصف مجموع الاكثر مع الاقل

٨ – مربع

¤ – ليس في المقالة التاسعة قضية تنص على ذلك إلا ان الدعوى حالة خاصة من القضية : مضروب سطحين متشابهين يكون عددا مربعا (الشكل الاول من المقالة التاسعة) .

كان د ومربع هب زوجين لإن ب يكون زوجا وان كان اج بج فردين كان د مربعا فردا ومربع هب زوجا لإنه لا يجتمع من عددين مربعين [فردين] عدد [٩] مربع . وان كان احدهما زوجا والآخر فردا كان د مربعا زوجا ووقع مربع هب في عدد غير صحيح ولم يُسَمَّ عددا مربعا **لان العدد ما رُكِّب من اعداد صحاح ولذلك يرى اصحاب الجبر ان يعبروا عمّا له جذر بمال[١] ليعم ماصحَّ من الاعداد المجذورة وما به[٢] كسور.**

12 **وفي وجود ثلثة اعداد مربعة مجموعها مربع** نأخذ ثلثة اعداد مربعة يكون اكثرها اكثر من مجموع الاقلّيّن ولتكن اب بج جد وننصف اد على ه ونجعل ز مضروب اب في بج و ح مضروب اب في جد فاقول ان مجموع عددي ز ح وهما مربعان مع مربع هد مثل مربع هب .



برهان ذلك | ان ضرب اب في بج وضرب اب في جد مثل ضرب اب في دب فضرب اب في دب مثل عددي ز ح ، ولكن ضرب اب في دب « هو مثل ضرب اد في دب ــ الذي هو[٣] » مثل ضرب هد في دب مرتين ــ ومربع دب . فضرب هد في دب مرتين ومربع دب مثل عددي ز ح . وضرب هد في دب مرتين ومربعا هد دب مثل مربع هب . فمربع هب مثل عددي ز ح المربعين مع مربع هد . ثم تعلم بما قدّمنا هل كلها ازواج او بعضها ازواج وبعضها افراد وبمثل هذا الطريق نجد اعدادا كثيرة مربعة مجموعها مربع . ١ب

13 ولإنا نحتاج فيما نأتي به من بعد الى عددين مربعين ضلع مجموعهما مربع[٥] والى عددين مربعين مجموعهما مربع وضلع احدهما مربع[٦] فإنا نبيّن وجود الاولين هكذا : كل عددين مربعين مجموعهما مربع فانه اذا ضرب احدهما في الآخر اربع مرات اجتمع اكثر المربعين اللذين ضلع مجموعهما مربع[٧] واذا أُخذ فضل ما بينهما وضرب في مثله اجتمع المربع

٩ — فرد ١ — مال ٢ — او شابه

٣ — « الذي هو مثل ضرب هب في دب مرتين »

٥ — $س^٢ + ص^٢ = ع^٤$ ٦ — $س'^٢ + ص'^٤ = ع'$

٧ — $ب^٢ + ج^٢ = د^٢$ $س^٢ = ٤ب^٢ج^٢$ $ص^٢ = (ب^٢ - ج^٢)^٢$

الأقل[٤]. مثال ذلك تسعة وستة عشر وهما اقل عددين مجموعهما مربع واذا ضرب احدهما في الآخر اربع مرات اجتمع خمس ماية وستة وسبعون وهي اكثر المربعين وضلعه مضروب ستة في اربعة والمربع الاقل تسعة و ربعون وضلعه سبعة ، وهو فضل ما بين تسعة وستة عشر وضلع مجموعهما ، وهو ستماية[٥] وخمسة وعشرون ، خمسة وعشرون[٦] .

14 واما وجود الآخرَين فعلى هذه الصفة : كل عدد ضلعه مربع اذا ضرب في ربع عددٍ ضلعهُ مربع اربعَ مرات اجتمع ذلك العدد نفسه[a] الذي ضلعه مربع ولكن الواحد مربعٌ ضلعه مربع والستة عشر مربع ضلعه مربع واذا ضرب الواحد في اربعة اربعَ مرات اجتمع ستة عشر وهي اكثر المربعين[b] وضلعه مربع ونأخذ فضل ما بين الواحد والاربعة
١٢٠٨ فنضربه في مثله فيكون | المربع الاقل[c] ومجموعهما خمسة وعشرون وهي اول عدد ينقسم بعددين مربعين ضلع احدهما مربع .

15 واذا اردنا وجود عدد آخر شبيه بخمسة وعشرين طلبنا عددين نسبة احدهما الى الآخر نسبة عدد مربع الى عدد مربع وفضل ما بينهما مربع ليكون مضروبه[d] في مثله مربعاً ضلعهُ مربع . واول عددين بهذه الصفة ثلثة واثنا عشر فان نسبة احدهما الى الآخر نسبة واحد الى اربعة وفضل ما بينهما مربع وهو تسعة والواحد والاربعة قسما الخمسة وفضل ما بينهما ثلثة واذا ضرب كل واحد من القسمين في ثلثة كان مجموع ذلك خمسة عشر مثل ما يجتمع من ضرب خمسة في ثلثة فخمسة عشر ضلع العدد الذي ينقسم بعددين مربعين ضلعُ احدهما مربع وهو مايتان وخمسة وعشرون واحد قسميه مضروب اثني عشر في مثلها وهو ماية واربعة واربعون والآخر مضروب تسعة في مثلها وهو مربع ضلعه مربع .

16 فان اردنا عددا ثالثا من هذه الاعداد وقد قدمنا انا اذا ضربنا عددا ضلعه مربع في ربع عدد ضلعه مربع اجتمع عدد ضلعه مربع نضرب ستة عشر في ربعها اربع مرات فيكون مايتين وستة وخمسين وضلعها مربع وهو ستة عشر ونأخذ فضل ما بين اربعة وستة عشر ونضربه في مثله فيكون مائة واربعة واربعون ومجموعهما اربعماية وضلعه مضروب اربعة

٤ – الأول
٥ – مايتان ٦ – في الهامش هنا جملة شرح خاطئة .
a – نفسه اي الذي ذكره في دعوى القضية وسميته ص ٤'
b – اي ما سميته ص ٤' c – اي ما سميته س ٢' d – مضروبه اي مضروب الفضل

في خمسة فهو عدد مربع ينقسم بعددين مربعين ضلع احدهما مربع وان ضربنا خمسة وعشرين في ستة عشر كان ايضاً اربعماية

17 وايضا اذا ضربنا خمسة في عدد نسبته الى ثلثة نسبة عدد مربع الى عدد مربع اجتمع عدد ينقسم بقسمين على نسبة عدد مربع الى عدد مربع وفضل ما بينهما مربع ومضروب احدهما في الآخر مربع وذلك مثل خمسة في اثني عشر فانه ستون وهي تنقسم باثني عشر وثمنية واربعين[7] وثمنية واربعون في اثني عشر ربع مرات مربع وهو الفان وثلثماية واربعة ٢٣٠٤ وفضل ما بينهما وهو ستة وثلثون .

٢٠ب وجملة القول انه اذا اخذ عددان ا مربعان لاحدهما ربع وعمل بهما ما تصيف وجد العدد المطلوب . مثال ذلك تسعة في ماية واربعة واربعين فان ضلع ذلك وهو ستة وثلثون مربع واذا جعل احد العددين ستة وثلثين والآخر تسعة وعمل بهما وبفضل ما بينهما مثل ما تقدم اجتمع عنهما الفان وخمسة وعشرون وانقسمت بمربعين ضلع احدهما مربع وهو الف ومايتان وستة وتسعون وضلعه ستة وثلثون والآخر سبع ماية وتسعة وعشرون وضلعه سبعة وعشرون . وعمل ذلك راجع الى ما عملناه في ستة عشر واربعة .

19 وفي وجود ذلك طريق آخر وهو ان مضروب اثنين وثلثين في ثمنية ضلعه مربع وهو ستة عشر فإن اخذ ربعه وهو اثنان وجعل احد العددين والآخر اثنين وثلثين حدث من ذلك الف وماية وستة وخمسون وانقسم بمأتين وستة وخمسين وبتسع ماية إلا أن طريق هذا الباب لا يجري على نظام بالطريق الذي ذكرناه وله طريق يلزم النظام في المربعات التي اضلاعها ازواج وذلك ان يجعل احد العددين عددا مجذورا له ربع والآخر ربعه واولها اربعة وستة عشر ،وتسعة وستة وثلثون،وستة عشر واربعة وستون . وقد بيّنا فيما تقدم انه لا يمكن ان يوجد عددان مربعان يكون مجموعهما مربعا فيكون ضلعاهما زوجي الزوج وانهما اذا كانا زوجين امكن ان يكون احدهما زوج الزوج والآخر زوج الفرد او زوج الزوج والفرد ، وان كان احدهما فردا امكن ان يكون الآخر زوج الزوج او زوج الفرد او زوج الزوج والفرد . ولذلك يكون مضروب احدهما في الآخر مرتين زوج الزوج والفرد ابدا .

[7] – اربعون

20 واقول ان كل عدد ينقسم بعددين مربعين فان ضعفه ينقسم بعددين مربعين . برهانه ان كل عددين مختلفين فان مجموع مربعيهما مثل مضروب احدهما في الآخر مرتين ومربع
٢٠٩ ا فضل ما بينهما مما يتبيّن في الوضع العددي في الشكل الخامس والتاسع من المقالة السابعة على الوجه الذي بيّن في المقالة الثانية من كتاب الاصول ، فيكون مضروب احدهما في الاخر اربع مرات وضعف مربع فضل ما بينهما مثل ضعف مجموع مربعيهما . ولكن مجموع مربعيهما ومضروب احدهما في الآخر مرتين مثل مربع مجموعهما فاذن ضعف مجموع مربعيهما يزيد على مربع مجموعهما بمربع فضل ما بينهما فلذلك كل عدد ينقسم بعددين مربعين فان ضعفه ينقسم بعددين مربعين فيكون ضلع الاعظم منهما مجموع ضلعي العددين المربعين الاولين وضلع الاقل فضل ما بين الضلعين . وعلى هذا الوجه[8] كل عدد ينقسم بعددين مربعين فان ضعفه ينقسم بعددين مربعين وضعف ضعفه وكذلك الى غير نهاية .

21 واقول ايضا ان كل عدد زوج ينقسم بعددين مربعين فان نصفه ينقسم بعددين مربعين ونصف نصفه وكذلك الى حيث يبلغ[7] . برهانه ان كل عدد زوج ينقسم بعددين مربعين فان كل واحد من قسميه يكون زوجا او فردا ولذلك يكون كل واحد من ضلعي قسميه زوجا او فردا فيكون مجموعهما زوجا ابدا . وفضل ما بينهما زوجا ولان كل عدد ينقسم بنصفين وبقسمين مختلفين فان مجموع مربعيهما يكون ضعف مربع نصف مجموعهما وضعف مربع نصف فضل ما بينهما لان نصف فضل ما بينهما هو فضل ما بين نصف مجموعهما وبين القسم الاكثر . ولذلك اذا جمع ضلعا عددين مربعين واخذ مربع نصف مجموعهما ومربع نصف فضل ما بينهما كان ذلك نصف مجموع المربعين . فاذاً كل عدد زوج ينقسم بعددين مربعين فان نصفه ايضا ينقسم بعددين مربعين وكذلك حتى تنتهي الى عدد غير صحيح
٢٠٩ب ويكون الضلع الاكثر نصف مجموع ضلعيهما والضلع الاقل نصف فضل ما بينهما ولذلك اذا كان العدد الذي ينقسم بمربعين فردا وقع في نصفه كسر ولم ينقسم بعددين مربعين لان العدد كما قلنا ما رُكّب من آحاد صحاح .

22 وبعد تقديم ما قدمناه نصير الى الغرض الذي نحوناه وهو ان نبيّن اذا فرض لنا عدد من الاعداد كيف نطلب عددا مربعا اذا زدنا عليه العدد المفروض ونقصناه منه كان ما بلغ

٨ — هذا الجهة س ; هذه الجهة ٧ — ما دام النصف زوجا وهذا بديهي

وما بقي عددين مربعين . فنتنزل وجود الاعداد المربعة الثلثة وهي الاقل والاوسط والاكثر على جهة التحليل فاقول ان العدد المربع الاوسط ينقسم بعددين مربعين لان المجموع منه ومن العدد المفروض مربع واذا زيد عليه فضل ما بين العدد الاوسط والعدد المفروض وهو كما قدمنا مربع اجتمع ضعف العدد الاوسط فهو اذاً زوج فقد انقسم مع ذلك بعددين مربعين فنصفه ايضاً ينقسم بعددين مربعين . فقد ظهر من ذلك ان كل عدد [مربع] يزاد عليه عدد مفروض وينقص منه فيكون المجتمع والباقي عددين مربعين فانه ينقسم بعددين مربعين واقول ان العدد المفروض هو ضعف العدد الذي يحيط به ضلعا العددين المربعين اللذين ينقسم بهما العدد الاوسط . برهان ذلك ان فضل المربع الاكثر على العدد المربع الاقل وهو ضعف العدد المفروض مثل مضروب مجموع ضلعيهما في فضل ما بينهما مما يتبين في الوضع العددي على الوجه الذي بُيّن في الشكل السادس من المقالة الثانية من كتاب الاصول ولكن مجموع ضلعي العددين المربعين اللذين ينقسم بهما العدد الاوسط هو الضلع الاكثر من ضلعي المربعين اللذين ينقسم بهما ضعف العدد الاوسط والضلع الاقل هو فضل ما بينهما كما بيّنا فيما تقدم ولذلك يكون العدد المفروض ضعف مضروب احد الضلعين في الآخر فالعدد المفروض ضعف العدد الذي يحيط به ضلعا المربعين اللذين ينقسم بهما العدد الاوسط وهو زوج فقد انعكس آخر[9] التحليل على انه متى فرض لنا عدد وطلب منا عدد مربع ان زدنا عليه ذلك العدد ونقصناه منه كان المجتمع والباقي مربعين وجب | ان يكون العدد المفروض زوجا والا يكون نصفه أوّل لانه يحيط به عددان مركبان والعدد الاول غير مركب والا يكون نصفه ايضا فردا وان كان مركبا لانه يحيط به عددان فردان ولا يمكن ان يكون مجموع مربعيهما مربعا فان كان العدد المفروض على احدى الحالين كان ما طلب محالاً .

2ظ فبقي ان يكون كلا العددين المربعين اللذين ينقسم بهما العدد المطلوب زوجا او يكون احدهما فردا والآخر زوجا وايهما كان فان مضروب ضلعيهما احدهما في الآخر مرتين وهو مثل العدد المفروض يكون زوج الزوج والفرد لان كل عدد زوج فان ضعفه يعده الاربعة وكل عدد يعده الاربعة فان العدد الذي يحدث من ضربه في عدد فرد يكون زوج الزوج والفرد فمتى فرض لنا عدد لم يكن زوج الزوج والفرد علمنا ان الذي طلب منا ممتنع

9 ــ اخر ، ص : اجزاء ، تقول وايا كانت القراءة فالتعبير غير واضح

الوجود لإنا قد بيّنا انه لا يمكن ان يوجد عددان مربعان كل واحد منهما زوج الزوج ويكون مجموعهما مربعا . فان كان احد ضلعي المربعين الزوجين زوج لزوج كان الآخر زوج الفرد او زوج الزوج والفرد وايهما كان فان مضروب احدهما في الآخر مرتين زوج الزوج والفرد . وان كان احد الضلعين فردا وكان الآخر احد اقسام الزوج كان مضروب احدهما في الآخر مرتين لا محالة زوج الزوج [والفرد] .

24 ولذلك اذا فرض لنا عدد هو زوج الزوج والفرد وطلب منا عدد مربع ان زدنا عليه ذلك العدد كان المجتمع مربعا وان نقصنا منه ذلك العدد كان الباقي مربعا فانا نأخذ نصفه ونأخذ الاعداد التي تعده فان كان منها عددان يكون مجموع مربعيهما مربعا فقد وجدنا مطلوبنا ونسمي هذين العددين من بين كل عددين يعدانه ويحيطان به وهما ضلعاه][10] قرينين .

٢١٠ب وان لم نجدهما كذلك كان الذي طلب غير ممكن. واول هذه الاعداد اربعة وعشرون فان نصفه اقل عدد من اعداد زوج الزوج والفرد فنأخذ كل عدد يعد اثني عشر وهو اثنان وستة وثلثة واربعة فقط ومجموع مربعي ثلثة واربعة مربع وهو خمسة وعشرون فخمسة وعشرون اقل عدد مربع اذا زيد عليه عدد كان المجتمع مربعا وان نقص منه ذلك العدد كان الباقي مربعاً ثم لا يوجد لاربعة وعشرين من الاضعاف ما له نصف يعده عددان قرينان حتى ننتهي الى مايتين واربعين فان نصفها وهو ماية وعشرون يعده ثمنية وخمسة عشر ومجموع مربعيهما مايتان وتسعة وثمنون وجذره سبعة عشر[1] واذا زيد عليه او نقص منه مايتان واربعون كان المجتمع الباقي مربعين .

25 فلإن مايتين واربعين يعدها عددان مربعان وهما اربعة وستة [عشر] نقسمها على كل واحد منهما فيخرج ستون وخمسة عشر . فلإن نسبة مايتين واربعين الى ستين كنسبة عدد مربع الى عدد مربع وهو نسبة الاربعة الى الواحد تكون هذه النسبة كنسبة مايتين وتسعة وثمنين الى مال مجذور ان زيد عليه او نقص منه ستون كان المجتمع والباقي مالين مجذورين فنقسم مايتين وتسعة وثمنين على اربعة فيخرج المال ويقع فيه كسر ولذلك لفظنا بالمال وايضا فنسبة مايتين واربعين الى خمسة عشر كنسبة ستة عشر الى واحد فنقسمها على ستة عشر فيخرج المال الذي اذا زيد عليه ونقص منه خمسة عشر كان المجتمع والباقي مالين مجذورين .

١٠ – في المخطوط عبارة . او يعده او ولا يحيطان به . كما قرأناها « او يعدانه ولا يحيطان به » ورأينا فيها زيادة لقارىء ما تفسد المعنى وقرأها الدكتور سعيدان على وجه حسن : « او يعداه او يحيطان به » .
١ – ثلثة وعشرون

26 وهذا طريق مطرد في وجود هذا النوع من المجذورات وهو انا اذا وجدنا مالا له جذر ان زدنا عليه عددا كان لما بلغ بجذر وان نقصناه منه كان للباقي جذر ثم فرض لنا عدد نسبته الى ذلك العدد كنسبة عدد مربع الى عدد مربع وجدنا المال الذي اذا زيد عليه العدد المفروض كان لما بلغ جذر وان نقص منه كان لما بقي جذر . مثال ذلك ان يكون المال الموجود خمسة وعشرين والعدد الذي يراد عليه وينقص منه اربعة وعشرين والعدد الذي فرض لنا ستة ونسبته الى اربعة وعشرين نسبة واحد الى اربعة فالمال المطلوب ربع الخمسة والعشرين ولذلك نقسمها على اربعة فيخرج ستة وربع فهي المال المجذور الذي اذا زيد عليه ونقص منه ستة كان المجتمع والباقي مجذورين . وكذلك اذا فرض لنا اربعة وخمسون ونسبتها الى اربعة وعشرين نسبة تسعة الى اربعة فيكون المال المطلوب ضعف وربع خمسة وعشرين فنضرب خمسة وعشرين في اثنين وربع فيخرج المال ستة وخمسين وربعا وجذره سبعة ونصف فان زدنا على المال اربعة وخمسين كان لما بلغ جذر وان نقصناها منه كان لما بقي جذر .

27 وان كان العدد المفروض سبعماية وعشرين كان لنصفه عددان قرينان احدهما تسعة والآخر اربعون لان مضروب احدهما في الآخر ثلثماية وستون ومجموع مربعيهما الف وستماية وأحد وثمنون وجذره احد واربعون ولان نسبة سبع ماية وعشرين الى ثمنين كنسبة تسعة الى واحد نقسم ألفا وستماية وأحد وثمنين على تسعة فيخرج المال الذي اذا زيد عليه ونقص منه ثمنون كان المجتمع والباقي مالين مجذورين . فأما الاربعون فان نسبتها الى «سبع ماية وعشرين »[٢] كنسبة واحد الى ثمنية [عشر] وليست كنسبة عدد مربع الى عدد مربع فليس يوجد من هذا الوجه مال يزاد عليه وينقص منه اربعون فيكون الزائد والناقص مالين مجذورين .

28 ومن وجه آخر فان نصف الاربعين انما يعده اثنان وعشرة وخمسة واربعة فقط وليس فيها عددان قرينان يكون مجموع مربعيهما مربعا . وعلامة ضلعي العـــدد المفروض هــل يمكن ان يكون مجموع مربعيهما مربعا أوْ لا يكون ذلك ان يقسم مربع الأقل على ضعف الاكثر فان كان ما يخرج | « جذرا لما بقي »[٣] سهُل وجود ما نريد والا تعذّر وامتنع

٢ — الى الف وستماية واحد وثمين
٣ — أو ما بقي له جذر

مثل مابة وعشرين فانه يحيط بها ثمنية وخمسة عشرَ واذا قسمنا مربع ثمنية على ضعف الخمسة عشر خرج الثان وبقي مربع الاثنين وهـــو اربعة فيتفقد ذلك في طلب هذه الاعداد . ومثل ثلثماية وستين فانه يحيط بها اربعون وتسعة بهذه الصفة وذلك ان مربع تسعة اكثر من اربعين واذا قسمناه على ضعف الاربعين خرج واحد وبقي واحد ولان مربع ما خرج مثل ما بقي يمكن وجود ما نريد .

29 وفي وجود الفرع الذي قدمنا ذكره من الاعداد طرق أخرَ مرجعها كلها الى خمسة وعشرين . منها انّا متى وجدنا عددا مربعا اذا زدنا عليه عددا مربعا ضلعه مربع كان للمجتمع جذر ثم قسمنا جذر مجموعهما على جذر جذر العدد المربع خرج لنا جذر مال اذا زدنا ضعف جذر العدد المربع على المال ونقصناه منه كان المجتمع والباقي مجذورين . واول هذه الاعداد تسعة فانّا ان زدنا عليه ستة عشر وله جذر ولجذرها جذر كان خمسة وعشرين واذا قسمنا جذرها وهو خمسة على جذر جذر ستة عشر خرج اثنان ونصف وهي جذر المال الذي ان زيد عليه ضعف جذر تسعة كان لما بلغ جذر وان نقص منه كان لما بقي جذر . ومن هذا النوع عدد اثني عشر فان مربعه الذي هو ماية واربعة واربعون اذا زدنا عليها احد وثمنين وهي عدد مربع ضلعه مربع اجتمع مايتان وخمسة وعشرون وهي عدد مربع ضلعه خمسة عشر فنقسمها على ثلثة وهي جذر تسعة فيخرج خمسة وهي جذر مال اذا زدنا عليه ونقصنا منه ضعف اثني عشر كان المجتمع والباقي عددين مجذورين .

30 ومنهـــا انّا نطلب عددين مربعين ضلع احدهمـــا مربع ومجموعهما مربع ووجوده ان نجعل احد العددين كما بيّنا فيما تقدم ربع عدد مجذور والآخر ذلك العدد المجذور ونضرب احدهما في الآخر | اربع مرات فيجتمع أحد العددين المربعين ونأخذ فضل ما بينهما فيكون ثلثة ارباع الاكثرw ويكون مضروبه في مثله العدد المربع الآخر ومجموعهما عدد مربع I ٢١٢ واذا قسمنا جذره على جذر العدد الاول خرج جذر المال ان زدنا عليه ذلك العدد ونصفه كان لما بلغ جذر وان نقصناه منه كان لما بقي جذر . مثال ذلك ستة عشر واربعة فانا نضرب احدهما في الآخر اربع مرات فيكون مايتين وستة وخمسين ونأخذ فضل ما بينهما فيكون ثلثة ارباع الاكثر وهو اثنا عشر ومربعها ماية واربعة واربعون ومجموعهما اربعماية وجذرها مجموع ستة عشر وربعها وهو عشرون اذا قسمناه على جذر ستة عشر خرجت خمسة وهي

w — اي ثلثة ارباع العدد المجذور المختار

جذر خمسة وعشرين واذا زدنا عليه مجموع ستة عشر ونصفها وهي اربعة وعشرون ونقصناه منها كان ما بلغ وما بقي عددين مجذورين .

31 ويتبيّن من ذلك انه اذا فرض لنا عدد لثلثيه جذر وجدنا المال الذي ان زدنا عليه ذلك العدد كان لما بلغ جذر وان نقصناه منه كان لما بقي جذر ووجود جذر ذلك المال المطلوب كما قدمنا ان نزيد على جذر ثلثي العدد المفروض ربعه* فيكون جذر المال المطلوب . وذلك ان لثلثي اربعة وعشرين وهو ستة عشر جذرا وهو اربعة واذا زيد عليه ربعه كان خمسة وهو جذر خمسة وعشرين . وهذه الاعمال ، فقد كان مرجعها الى خمسة وعشرين واربعة وعشرين كما بيّناه في اول الامر ومن تأملها وقف على عللها ان شاء الله .

32 واقرب هذه الوجوه كلها ان نأخذ اي عدد شئنا ونزيد عليه ربعه وهو الاول ونزيد على ما اخذناه نصفه ونضربه فيما اخذناه فيكون الثاني ، فاذا زدنا الثاني على مربع الاول كان لما بلغ جذر وان نقصناه منه[4] كان لما بقي جذر مثال ذلك ان نأخذ ثمنية ونزيد عليها ربعها فيكون عشرة وهو الاول ونزيد على ثمنية نصفها ونضرب ما بلغ في ثمنية فيكون ستة وتسعين وهي الثاني واذا زدنا هذا الثاني على مربع الاول كان لما بلغ جذر وان نقصناه ٢ب
منه كان لما بقي جذر .

33 وقد ينشد في صناعة الجبر عن مال له جذر واذا زيد عليه عشرون كان لما بلغ جذر وان نقص منه عشرون كان لما بقي جذر وذلك يتعذر وجوده في عدد صحيح والوجه في معرفته ان نضرب عشرين في ستة وثلثين وهي عدد مربع فيجتمع سبعماية وعشرون فنطلب عدداً[5] ان زدنا عليه سبع ماية وعشرين اجتمع مربع وان نقصناها منه كان الباقي مربعا وهو الف وستماية وأحد وثمنون ١٦٨١ ووجودها يكون بالعمل الذي قدمناه فنقسمها على ستة وثلثين فيخرج المال المطلوب على ان هذا الطريق غير محصور وهو شبيه بالاستقراء اذ كانت الاعداد المربعة بلا نهاية ولذلك ربما اتفق ما نطلبه وربما تعذر

34 والطريق الصناعي في ذلك ان نأخذ نصف العشرين ونضربه في مثله فيكون ماية ونطلب مالا له جذر ولجذره جذر اذا زدناه على ماية كان لما يجتمع جذر . وانما يتفق لنا ذلك في

* - اي ربع ثلثي العدد
٤ — ذلك ٥ — عدد

العدد الذي قدمناه وهو الف وستماية واحد وثمنون اذا جعلناها اجزاء من ستة عشر ليكون احد وثمنون خمسة وجزءا من ستة عشر وجذرها تسعة اجزاء من اربعة وهي جذر ستة عشر فهي اثنان وربع وجذرها واحد ونصف فنزيد خمسة وجزءا من ستة عشر على ماية فيكون جذر الجميع عشرة وربعا . وذلك ان جذر الف وستماية واحد وثمنين احد واربعون وهي اجزاء من جذر ستة عشر واذا قسمناها عليه خرج عشرة وربع فنقسمها على واحد ونصف فيخرج ستة ونصف وثلث فهي جذر المال المطلوب والمال ستة واربعون [وخمسة وعشرون] جزءا[6] من ستة وثلثين فنزيد عليه عشرين فيبلغ ستة وستين وخمسة وعشرين جزءا من ستة وثلثين وجذره ثمنية وسدس وننقص من المال عشرين فيبقى ستة وعشرون وخمسة وعشرون جزءا من ستة وثلثين وجذره[7] خمسة وسدس . فقد تبين من ذلك انه اذ فرض لنا عدد وضربنا نصفه في مثله وحفظناه وطلبنا عددا له جذر ولجذره جذر اذا ردناه على ما حفظنا كان لما بلغ جذر فانا نجد المطلوب .

١٢١٣ ويتصل مما قدمنا ان نذكر جملة من خواص | الاعداد التي ينقسم كل واحد منها بعددين مربعين اذا[8] ضرب في عدد ينقسم بعددين مربعين كان احدهما مربعا أو[8] كان كل واحد منهما مربعاً او لم يكن واحد منهما مربعاً فان ذلك مما يوضح **المقدمة التي قدمها ذيوفنطس للمسئلة التاسعة عشرة[9] من المقالة الثالثة من كتابه في الجبر** وينتفع به في غيرها من المسائل . واول ذلك ان نقول كل عدد ينقسم بعددين مربعين فان مربعه ينقسم بعددين مربعين لان مضروب احدهما في الآخر اربع مرات يكون احد مربعي مربع ذلك العدد وضلعه مضروب جذر احدهما في جذر الآخر مرتين وضلع المربع الآخر فضل ما بين قسمي ذلك العدد .

36 وكذلك يكون حال كل عدد ينقسم بعددين مسطحين متشابهين . مثال ذلك عشرة فانها تنقسم باثنين وثمنية وهما مسطحان متشابهان فتنقسم الماية بعددين مربعين احدهما الاكثر اربعة وستون وهي مضروب ثمنية في اثنين اربع مرات والاقل[1] ستة وثلثون وهي مربع فضل ما بينهما .

37 فان [كان] العدد الذي ينقسم بعددين مربعين مربعا مثل خمسة وعشرين فان مربعها وهو ستماية وخمسة وعشرون ينقسم بعددين مربعين مرتين لان مضروب تسعة في خمسة

٦ – وجزء ٧ – وجذر
٨ – اذ ٩ – عشر ١ – والاكثر

وعشرين مربع وكذلك مضروب ستة عشر في خمسة وعشرين فينقسم ستماية وخمسة وعشرون بمربعين احدهما اربعماية والآخر مايتان وخمسة وعشرون . وينقسم ايضا بمربعين آخرين على الطريق الذي قدمنا وذلك ان مضروب احد قسمي خمسة وعشرين في الآخر اربع مرات يكون مربعا وهو خمسماية وستة وسبعون ويكون المربع الآخر مربع فضل ما بينهما وهو تسعة واربعون .

38 فان ضربنا عددا ينقسم بعددين مربعين مرة واحدة في عدد ينقسم بعددين مربعين مرة واحدة انقسم العدد المركب منهما بعددين مربعين مرتين . مثاله ان خمسة مركبة من واحد واربعة وثلثة عشر مركبة من اربعة وتسعة ومضروب احدهما في الآخر خمسة وستون فهي تنقسم بعددين مربعين مرتين لانه من البيّن ان خمسة في ثلثة عشر هو خمسة في اربعة وخمسة في تسعة وان خمسة في اربعة هو اربعة في اربعة وواحد في اربعة وخمسة

٢١١ب في تسعة هو اربعة | في تسعة وواحد في تسعة لان الخمسة ينقسم باربعة وبواحد وثلثة عشر ينقسم باربعة وبتسعة ويكون اضلاع هذه المربعات اثنين واربعة وثلثة وستة . ولان نسبة اثنين الى اربعة كنسبة ثلثة الى ستة يكون مضروب اثنين في ستة مثل مضروب اربعة في ثلثة ومضروب ثلثة في اربعة مرتين مثل مضروب اثنين في ستة مرتين ، ولان مضروب اثنين في ستة مرتين مع مربع فضل ما بينهما مثل مجموع مربعي اثنين وستة ، ولكن مضروب اثنين في ستة مرتين مثل مضروب ثلثة في اربعة مرتين ، ومضروب ثلثة في اربعة مرتين مع[2] مجموع مربعي ثلثة واربعة مثل مربع مجموع ثلثة واربعة ، يكون مربع فضل ما بين اثنين وستة مع مربع مجموع ثلثة واربعة مثل مجموع مربعات اثنين وثلثة واربعة وستة وهي خمسة وستون . فلذلك ينقسم خمسة وستون بمربعين ضلع احدهما مجموع ثلثة واربعة وضلع الآخر فضل ما بين اثنين وستة ، مرة اولى ؛ وينقسم مرة اخرى بمربعين ضلع احدهما فضل ما بين ثلثة واربعة ، وضلع الآخر مجموع اثنين وستة فينقسم خمسة وستون مرة اولى بتسعة واربعين وستة عشر ومرة اخرى بواحد واربعة وستين . وكذلك ينقسم مضروب كل عددين ينقسم كل واحد منهما بعددين مربعين احدهما في الآخر .

39 فان ضُرب خمسة وستون وهي تنقسم بعددين مربعين مرتين في أحد وستين وهي

٢ – مثل

تنقسم بعددين مربعين مرة واحدة كان ذلك ثلثة الاف[3] وتسعماية وخمسة وستين ٣٩٦٥ وهي تنقسم بعددين عددين مربعين اربع مرات لانه اذا ضرب عدد ينقسم بعددين مربعين مرة في عدد ينقسم بعددين مربعين مرة تولّد من ذلك عدد ينقسم بعددين مربعين مرتين . فاذا كان احدهما ينقسم بعددين مربعين مرتين وجب ان يكون مضروب احدهما في الآخر ينقسم بعددين مربعين اربع مرات وذلك يظهر هكذا : وهو ان نضرب كل واحد من ضلعي خمسة وعشرين وستة وثلثين في كل واحد من اضلاع مربعات[4] خمسة وستين فيحدث من ذلك اربعة اعداد متناسبة على نسبة خمسة الى ستة واربعة اخرى على هذه النسبة ونضعها على الرسم فيكون خمسة في ثمنية واربعين مثل ستة في اربعين

٤٢	٣٥	٢٤	٢٠	٤٨	٤٠	٦	٥

وعشرون في اثنين واربعين مثل اربعة وعشرين في خمسة وثلثين . فاذا عملنا في ذلك على ٢١٤ ا
نحو ما عملنا فيما تقدم وهو ان نأخذ فضل ما بين خمسة وثمنية واربعين وهو ثلثة واربعون ونجمع ستة مع اربعين فيكون ستة واربعين وهي قرين ثلث واربعين وينقسم الاصل بمربعيهما مرة ونجمع خمسة مع ثمنية واربعين فيكون ثلثة وخمسين ونأخذ فضل اربعين على ستة فيكون اربعة وثلثين وهي قرين ثلثة وخمسين فينقسم الاصل بمربعي ثلثة وخمسين واربعة وثلثين مرة اخرى فيكون قد انقسم الاصل بمربعين مرتين . وكذلك يعمل بالاربعة الاعداد الباقية وهي عشرون واربعة وعشرون وخمسة وثلثون واثنان واربعون فينقسم مضروب خمسة وستين في احد وستين بعددين عددين مربعين اربع مرات

40 وان ضربنا خمسة وستين في خمسة وعشرين وهي عدد مربع ينقسم بقسمين مربعين فانه يجتمع منه الف وستماية وخمسة وعشرون وينقسم بقسمين قسمين مربعين ست مرات اربعا منها على نحو ما بيناه ومرة خامسة من مضروب كل واحد من تسعة واربعين وستة عشر في خمسة وعشرين ومرة سادسة من مضروب كل واحد من اربعة وستين وواحد في خمسة وعشرين .

41 فان ضربنا خمسة وستين في مثلها اجتمع اربعة الف[3] ومايتان وخمسة وعشرون وهي

٣ — كتبت الالف بشكل الف اي بتقدير الف الجمع وهي كتابة جائزة في الالف ، دراهم ، اذ لم يقع التباس في المعنى .
٤ — المربعات

تنقسم بعددين عددين مربعين اربع مرات وذلك يظهر على ما يبينه . وطريق معرفة ذلك ان نعمل بخمسة وستين كما عملنا بالخمسة وذلك ان نقسمها باربعة وستين وبواحد ونضرب ضعف ضلع اربعة وستين في ضلع الواحد فيكون ستة عشر وهي ضلع القسم الاقل من مربع خمسة وستين ونأخذ فضل ما بين اربعة وستين وواحد وهو ثلث وستون وهي قرين ستة عشر . وكذلك نعمل ستة عشر وتسعة واربعين فيخرج ضلعا المربعين في المرة الثانية ثلثة وثلثين وستة وخمسين فقد قسمنا مربع خمسة وستين بعددين مربعين مرتين ونقسمه ايضا مرتين كما قسمنا مضروب خمسة في ثلثة عشر وذلك ان نضرب كل واحد من ضلع واحد ومن ضلع اربعة وستين في كل واحد من ضلع ستة عشر وضلع تسعة واربعين فنضرب واحدا في اربعة فيكون اربعة ، وثمنية في اربعة فيكون اثنين وثلثين ، ونضرب
٢١ب واحدا في سبعة فيكون | سبعة ونضرب ثمنية في سبعة فيكون ستة وخمسين . فمربعات هذه الاعداد اذا جمعت كانت مثل مربع خمسة وستين كما كانت مربعات اثنين وثلثة واربعة وستة مجموعةً مثل خمسة في ثلثة عشر . ولان نسبة اربعة الى سبعة كنسبة اثنين وثلثين الى ستة وخمسين يحدث من ذلك اربعة اعداد مربعة مجموع كل اثنين منها مربع خمسة وستين ، وذلك ان نجمع اربعة مع ستة وخمسين فيكون ستين ونأخذ فضل اثنين وثلثين على سبعة وهو خمسة وعشرون فيكون قرين ستين ونأخذ فضل ستة وخمسين على اربعة فيكون اثنين وخمسين ونجمع اثنين وثلثين وسبعة فيكون تسعة وثلثين وهي قرين اثنين وخمسين . ونضع ذلك على هذا الرسم. ومربع الخمسة والستين معما ينقسم به من المربعات **هو الذي قدّمه ذيوفنطس في المسئلة التي ذكرناها**[٧] وهي وجود اربعة اعداد اذا زيد كل واحد منها على مربع مجموعها كان لما بلغ جذر وان نقص منه كل واحد منها كان لما بقي جذر .

٣٩٦٩	٦٣	١٦	٢٥٦
٣٦٠٠	٦٠	٢٥	٦٢٥
٣١٣٦	٥٦	٣٣	١٠٨٩
٢٧٠٤	٥٢	٣٩	١٥٢١

42 وقد تتيح[٥] هذه المقدمة طريقاً يوجد به اربعة اعداد مختلفة يكون مجموعها مربعا ومجموع كل اثنين منها مربعا . فقد ينبغي للانسان ان يكون غرضه في المقدمات التي يعطاها ابتداع ما ينتج منها دون الاشتغال بزيادتها وتكثيرها فكم من نتائج ومطلوبات في المقدمات التي

٧ — انظر المقطع ٣٥
٥ — سح . س . تنتج

اعطاناها نيقوماخوس في صناعة العدد . وفي الاصول التي ضمنها اقليدس مقالاته العددية الثلث ونقله اليها من الارثماطيقي وبرهن عليها من جهة الخطوط ثم ختمها بوجود العدد التام الذي هو اجل الاغراض ، وجمه من الاعداد الارواج لان اصحاب الارثماطيقي قسموا العدد الزوج قسمة اخرى الى ثلثة انواع زائد وناقص وتام . وكان ينبغي لهم الا يخصوه بهذه الاقسام وقد وجد في العدد الفرد زائد وناقص . ولذلك وقع للسائلين سؤال
٢١٥ ا هل يوجد عدد تام من الاعداد الافراد ام لا . وقد يقع سؤال اخر | جليل وهو هل يجوز ان يوجد في بعض العقود دون بعض . فان المفسرين لكتاب الارثماطيقي قالوا : العدد التام موجود في كل عقد من العقود ولكن الناظرين في هذا الكتاب كثير[٦] والمستقصين لمعانيه[٧] اقل القليل والانسان اذا شهر بصناعة من الصناعات وجب ان يشرف على جزئياتها ما امكن ، ولا يقتصر على كلياتها فقط . فان اوائل كل صناعة هي كليات وكذلك جزئيات .

تم ولله الحمد والمنة .

عورض بالاصل

٦ – س : كثر
٧ – لمعانيها س : لمعانيها

$$0 < 9k^4 - 14k^2 + 1 \qquad \left(3k^2 - \frac{7}{3}\right)^2 - \frac{49}{9} + 1 > 0$$

$$\left(3k^2 - \frac{7}{3}\right)^2 > \frac{40}{9} \qquad 3k^2 - \frac{7}{3} > \frac{\sqrt{40}}{3} \quad \text{ou} \quad \frac{7}{3} - 3k^2 > \frac{\sqrt{40}}{3}$$

$$k^2 < \frac{7 - \sqrt{40}}{9} \quad \text{et} \quad k^2 > \frac{7 + \sqrt{40}}{9}$$

$$k < \frac{\sqrt{7 - \sqrt{40}}}{3} \quad \text{et} \quad k > \frac{\sqrt{7 + \sqrt{40}}}{3}$$

$$\frac{9k^4 - 14k^2 + 1}{16k^2} < \frac{1}{4} \qquad 9k^4 - 14k^2 + 1 < 4k^2 \qquad 9k^4 - 18k^2 + 1 < 0$$

$$(3k^2 - 3)^2 - 8 < 0 \quad \left\{ \begin{array}{lll} 3k^2 - 3 < \sqrt{8} & \text{d'où} \quad k^2 < \dfrac{3 + \sqrt{8}}{3} & \text{pour } k > 1 \\ 3 - 3k^2 < \sqrt{8} & \phantom{\text{d'où}} \quad k^2 > \dfrac{3 - \sqrt{8}}{3} & \text{pour } k < 1 \end{array} \right.$$

$$\frac{\sqrt{9 - 3\sqrt{8}}}{3} < k < \frac{\sqrt{9 + 3\sqrt{8}}}{3}$$

On pourra prendre par exemple, $0.240 < k < 0.273$

$1.217 < k < 1.393$

par exemple, $k = 0.15$, $k = 1.25$.

Note: On trouve dans Diophante des exemples d'inégalités du second degré V, 30, 10. Voir la discussion qu'en fait Heath, *Diophantus*, *op. cit.*, pp. 60—65.

D'autre part la décomposition du trinôme du second degré en un carré de *binôme* du premier degré est explicitement attribuée à Diophante par al-Karajī dans *al-Fakhrī* (Caire Ms 6663, f. 22a, 24a) encore qu'on n'en voit pas d'exemple dans l'Arithmétique de Diophante, éd. Tannery.

La considération que nous avons faite que la racine de $\left(3k^2 - \frac{7}{3}\right)^2$ est, suivant le cas $3k^2 - \frac{7}{3}$ ou $\frac{7}{3} - 3k^2$ est également faite par al-Karajī par exemple dans *ʿIlal ḥisāb al-jabr w'al-muqābala*, MS Bodl. Oxford. I. 986, 3, f. 4a, l. 1, et *al-Fakhrī*, Caire MS 6663, f. 24a, l. 1.

4ème siècle H. comme le 3ème d'ailleurs furent en effet une époque de recherche active où l'esprit critique – que l'on voit poindre ici – avait tous ses droits. On pense à ces réunions de penseurs et philosophes des 3ème et 4ème siècles, où chose inouïe, des hommes de races, de confessions, d'appartenances différentes mettaient leurs livres révélés de côté, pour discuter au nom de la raison.[6] Abū Jaʿfar se fait ici l'écho des critiques soulevées à propos de la théorie des nombres et des recherches entreprises. Le fait qu'il nous propose de trouver quatre nombres dont la somme est un carré et qui ajoutés deux à deux donnent un carré signifie sinon qu'il en avait la solution du moins qu'il était sur la voie de la recherche. Ce joli problème est digne de figurer dans des commentaires sur Diophante comme en ont écrit al-Būzjānī ou al-Samaw'al. La solution que nous en donnons à la manière de Diophante montre que le problème n'est pas au-dessus des possibilités d'Abū Jaʿfar. Il s'agit de trouver des nombres possédant les propriétés énoncées. On peut voir une solution par Fermat du système $ax + b = \square$, $cx + d = \square$, $ex + f = \square$, dans T. L. Heath, *Diophantus*, p. 321.

Problème - Trouver quatre nombres dont la somme est un carré et qui, additionnés deux à deux donnent des carrés.

Solution: Soient a, b, c, d, ces quatre nombres. Nous faisons $a = x^2$, $b = -2mx + m^2$, $c = 2nx + n^2$, $d = 2px + p^2$, de sorte que $a + b$, $a + c$, $a + d$ sont des carrés. La somme $a + b + c + d = x^2 + 2(-m + n + p)x + m^2 + n^2 + p^2$ sera identique à un carré, si nous prenons $(m - n + p)^2 - (m^2 + n^2 + p^2)$ ou $-2mn - 2mp + 2np = 0$, $m = \dfrac{np}{n+p}$, égalité vérifiée par une infinité de solutions (m, n, p) entières ou rationnelles. Reste à égaler $b + c$, $b + d$, $c + d$, à des carrés

$$2(n - m)x + n^2 + m^2, \quad 2(p - m)x + p^2 + m^2, \quad 2(p + n)x + p^2 + n^2$$

Nous réduisons la difficulté en prenant deux de ces trois expressions égales. Il suffit de prendre $n = p$. Faisons par exemple, $n = p = 1$, d'où $m = \dfrac{1}{2}$,

$$b + c = x + \frac{5}{4}, \quad b + d = x + \frac{5}{4}, \quad c + d = 4x + 2.$$

Il s'agit de rendre $4x + 5$ et $4x + 2$ carrés.

Posons $\begin{cases} 4x + 5 = u^2 \\ 4x + 2 = v^2 \end{cases}$

D'où $x = \dfrac{u^2 - 5}{4}$ où u est rationnel, et $u^2 - v^2 = 3$, u et v rationnels.

Posons $\begin{cases} u + v = 3k \\ u - v = \dfrac{1}{k}, \quad k \text{ rationnel.} \end{cases}$

Ainsi $u = \dfrac{3k^2 + 1}{2k}$, $v = \dfrac{3k^2 - 1}{2k}$, $x = \dfrac{9k^4 - 14k^2 + 1}{16k^2}$.

Condition $b = -x + \dfrac{1}{4} > 0$, pour $0 < x < \dfrac{1}{4}$

6. Voir al-Dabbi, *Bughyat al-multamis fi tārīkh rijāl ahl al-andalus* (Caire, 1967), p. 155.

entre les textes grecs de Diophante tels qu'ils ont été connus des Arabes et ceux qui sont conservés de nos jours. En même temps elle confirme l'affirmation émise par Roshdi Rashed que le livre III du texte est conforme au livre III de la traduction arabe.[2]

Texte 43 Cette proposition pourrait fournir un moyen de *trouver quatre nombres dont la somme est un carré et qui additionnés deux à deux donnent des carrés*. Car il convient de tirer des propositions préliminaires, leurs conséquences immédiates sans chercher à augmenter le nombre de ces propositions.[3] Que de résultats et de questions posées dans les propositions que Nicomaque nous a données dans la théorie des nombres *(ṣinā-ᶜat al-ᶜadad)* et dans les Eléments qu'Euclide a transférés de la théorie des nombres à ses trois livres arithmétiques, éléments qu'il a démontrés au moyen de segments et qu'il a couronnés. par la recherche du nombre parfait qui est le but suprême. Euclide a placé les nombres parfaits dans la catégorie des nombres pairs car les arithméticiens ont réparti les nombres pairs en trois classes: surabondants, déficients et parfaits. Il n'auraient pas dû caractériser les nombres pairs par cette division puisqu'on a trouvé des nombres impairs surabondants et déficients. On s'est demandé de même s'il existe un nombre parfait impair. Une autre question importante que l'on peut se poser c'est si le nombre parfait peut se trouver dans certains *ᶜuqūd*[4] et pas dans d'autres. Car les commantateurs du livre de l'Arithmètique[5] ont dit qu'il y a un nombre parfait dans chacun des *ᶜuqūd*. (Mais tant s'en faut) car les lecteurs de ce livre sont nombreux et ceux qui approfondissent ses notions sont très rares. Or les personnes qui acquièrent un renom dans la science *(ṣināᶜat)* ne doivent pas se contenter d'en connaître les généralités mais être maîtres aussi de ses plus petits détails. Le début de chaque science est généralités la perfection en est dans les minuties.

Observation. L'intérêt du langage précédent est évident il évoque un climat. L'attitude d'Abū Jaᶜfar qui n'est pas celle d'un isolé est que le rôle de savant ne doit pas se limiter à celui de transmettre. Bien des questions laissées sans réponse attendent de lui leurs solutions. Le

2. Voir l'important article de Roshdi Rashed, "Les travaux perdus de Diophante," *Revue d'Histoire des Sciences*, 27 (1974), 99-122, p. 105; 28 (1975), 3-30.

3. Il est possible que la suppression, par un copiste, de la négation *lā'* avant le mot convient ait modifié le sens de la phrase.

4. *ᶜaqd*, pl. *ᶜuqūd* signifie ici la classe des unités, celle des dizaines, des centaines, des milliers, des dizaines de mille. . . Dans l'Arithmétique de Nicomaque il est dit que dans chaque classe jusqu'à celle des mille, il y a un nombre parfait et un seul 6, 28, 496, 8128 (*Kitab al-madkhal ilā ᶜilm al-ᶜadad*, trad. Thābit b. Qurra, W Kutsch, S.J., (Beyrouth, 1958) pp. 38-29).

5. Il s'agit évidemment de l'Arithmétique de Nicomaque qui connut chez les Grecs et les Arabes un crédit considérable. Jamblique (283-330) énonça qu'il y avait dans chaque classe de nombres, unités, dizaines, etc. . . . , jusqu'à l'infini un nombre parfait et un seul, affirmation erronée (Voir Dickson, *op. cit.*, vol. I, p. 4). On doit à Thābit b. Qurra un mémoire sur les nombres parfaits (F. Woepcke, *Jour. As.*, 20 (1852), 420-9). Le 5ème nombre parfait 33550336 se trouve mentionné dans un ms. latin daté en partie de 1456, en partie de 1461 (Dickson, *op. cit.*, p. 6).

Et nous avons

$$
\begin{aligned}
xy &= (ac - bd)^2 + (bc + ad)^2 \\
&= (ac + bd)^2 + (bc - ad)^2 \\
&= (a'c - b'd)^2 + (b'c + a'd)^2 \\
&= (a'c + b'd)^2 + (b'c - a'd)^2
\end{aligned}
$$

Texte 40 Si un nombre x se décompose en une somme de 2 carrés de deux manières différentes et si un carré y^2 se décompose d'une seule manière, leur produit se décompose de six manières différentes en une somme de 2 carrés.

$$x = a^2 + b^2 = a'^2 + b'^2 \qquad y^2 = c^2 + d^2$$

Aux quatre décompositions déjà vues s'ajoutent :

$$
\begin{aligned}
xy^2 &= a^2 (c^2 + d^2) + b^2 (c^2 + d^2) \\
xy^2 &= a'^2 (c^2 + d^2) + b'^2 (c^2 + d^2)
\end{aligned}
$$

Ex. : 65 25 etc. . . .

Texte 41 (Si un nombre est une somme de deux carrés de deux manières, son carré l'est de quatre manières).

$$x = a^2 + b^2 = c^2 + d^2$$

D'où $\quad x^2 = (2ab)^2 + (a^2 - b^2)^2$,

et $\quad x^2 = (2cd)^2 + (c^2 - d^2)^2$.

On a aussi $\quad x^2 = (ad + bc)^2 + (ac - bd)^2$,

$\quad x^2 = (ac + bd)^2 + (ad - bc)^2$.

La question est exposée dans le texte sur $65 = 8^2 + 1^2 = 4^2 + 7^2$, et les résultats groupés dans un tableau.

$$
\begin{aligned}
x^2 &= 16^2 + 63^2 = 60^2 + 25^2 \\
x^2 &= 56^2 + 33^2 = 39^2 + 52^2
\end{aligned}
$$

256	16	63	3969
625	25	60	3600
1089	33	56	3136
1521	39	52	2704

L'auteur ajoute: La décomposition de 65^2 en somme de deux carrés est ce que Diophante a placé en tête de la question *(qaddama)* que nous avons rappelée : Trouver quatre nombres qui ajoutés successivement au carré de leur somme donnent des carrés et qui, retranchés du carré de leur somme, donnent des carrés.

Observation: Nous avons rendu le mot *qaddama* par placer en tête. Ce mot signifie également donner en lemme et l'expression utilisée par l'auteur dans le paragraphe 35 rend clairement cette dernière signification : *al-muqaddima allati qaddamaha Dyhofantus lil'-mas'alat al-tāsiᶜata ᶜashara*. Dans le texte établi par Tannery la décomposition de 65^2 en somme de deux carrés de quatre manières est rapportée dans le texte de la prop. 19 du livre III, mais en lemme. Si notre interprétation du mot *qaddama* est exacte, cette circonstance montrerait les différences

Quelques propriétés des nombres qui se décomposent en sommes de
Texte *carrés, utiles dans certaines questions et éclairant le lemme dont Diophante*
35 *a fait précéder la proposition III, 19 de son Algèbre.*

Si x est une somme de deux carrés son carré est aussi une somme de deux carrés.

$$x = a^2 + b^2, \quad x^2 = (2ab)^2 + (a^2 - b^2)^2.$$

Texte Si x est une somme de deux nombres plans semblables, son carré
36 est une somme de deux carrés.

$x = ab + cd$ avec $a:b = c:d$. Donc $(ab)(cd)$ est un carré (*Euclide*, IX, I). $x^2 = 4(ab)(cd) + (ab - cd)^2$.

Texte Si un carré se décompose en une somme de 2 carrés, son carré se
[redacted] décompose en une somme de 2 deux carrés de deux manières différentes.

$$x^2 = a^2 + b^2 \text{ donne } x^4 = (2ab)^2 + (a^2 - b^2)^2,$$
$$x^4 = a^2(a^2 + b^2) + b^2(a^2 + b^2).$$

Ex. : $25 = 3^2 + 4^2$, $625 = 4 \cdot 9 \cdot 16 + (4^2 - 3^2)^2$,
$625 = 9 \cdot 25 + 16 \cdot 25.$

Texte Si deux nombres sont des sommes de 2 carrés leur produit est une
38 somme de 2 carrés de deux manières différentes.

Si $x = a^2 + b^2$ et $y = c^2 + d^2$,
on a $xy = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

Mais $ac:ad = bc:bd$ donc $ac.bd = ad.bc$ (*Euclide* VII, 19). Par suite on peut écrire :

$$xy = |ac - bd|^2 + (ad + bc)^2 \quad \text{et}$$
$$xy = (ac + bd)^2 + |ad - bc|^2.$$

Ex. : $5 \cdot 13 = (1^2 + 2^2) \cdot (2^2 + 3^2)$.
$5 \cdot 13 = 4^2 + 7^2$.
$5 \cdot 13 = 8^2 + 1^2$.

Texte Si deux nombres se décomposent en une somme de deux carrés,
39 l'un de deux manières différentes, l'autre d'une seule manière, leur produit se décompose de quatre manières en somme de 2 carrés.

On a $x = a^2 + b^2 = a'^2 + b'^2$, $y = c^2 + d^2$.
$$xy = a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2$$
$$xy = a'^2c^2 + b'^2c^2 + a'^2d^2 + b'^2d^2$$

Les produits de c et d par les termes a, b; a', b' sont huit nombres dont le rapport (deux à deux) est celui de c à d.

ac	bc	ad	bd	$a'c$	$b'c$	$a'd$	$b'd$

($ac : ad = bc : bd$ donne $ac \cdot bd = ad \cdot bc$).

x tel que $x^2 \pm a = \square$ prendre $x = b + \frac{b}{4}$.

$\left[\text{En effet, } \left(\frac{5b}{4}\right)^2 \pm \frac{3b^2}{2} = \frac{25b^2 \pm 24b^2}{16}\right]$.

Ex. : $a = 24$, $\frac{2}{3}a = 16 = 4^2$, $x = 4 + \frac{1}{4} \cdot 4 = 5$.

Texte 32 La méthode la plus simple pour trouver un (x,a) tel que $x^2 \pm a = \square$ est de choisir un nombre arbitraire t et de poser $x = \frac{5t}{4}$, $a = \frac{3t^2}{2}$. Alors $x^2 \pm a = \square$.

Ex. : $t = 8$, $x = 10$, $a = t \cdot \frac{3t}{2} - 96$. On a bien $102 \pm 96 = \square$.

Texte 33 On recherche en algèbre (*ṣinā^cat al-jabr*) x tel que $x^2 \pm 20 = \square$, équation impossible pour x entier. (Pour x fractionnaire) on considère le produit $20 \cdot 36 = 720$. Il est facile de trouver un carré d'entier u tel que $u^2 \pm 720 = \square$, $u = 41$, $41^2 \pm 720 \quad \square$. Par division par 36, $\left(\frac{41}{6}\right)^2 \pm 20 = \square$.

Texte 34 Cependant cette méthode est une méthode d'essais qui peut donner ou ne pas donner de résultat.

La méthode régulière (*ṣinā^cī* artisanal) pour calculer x tel que $x^2 \pm a = \square$ où a est donné, est de trouver un u tel que $(u^2)^2 + \left(\frac{a}{2}\right)^2 = \square$.

Soit $(u^2)^2 + \left(\frac{a}{2}\right)^2 = b^2$, d'où $u^2 + \left(\frac{a}{2u}\right)^2 = \left(\frac{b}{u}\right)^2$.

On a $x = \frac{b}{u}$. En effet, $x^2 \pm a = u^2 + \left(\frac{a}{2u}\right)^2 \pm a = \left(u \pm \frac{a}{2u}\right)^2$.

(Les explications sont données sur $x^2 \pm 20 = \square$).

On a $\frac{1681}{16} = 100 + \frac{81}{16}$ ou $\left(\left(\frac{3}{2}\right)^2\right)^2 + \left(\frac{20}{2}\right)^2 = \left(\frac{41}{4}\right)^2$.

Alors $x = \frac{41}{4} : \frac{3}{2} = \frac{41}{6}$ (ou $6\,\frac{1}{2}\,\frac{1}{3}$) etc. . . .

Note préparatoire

Diophante a montré que :

1. Tout carré, ou toute somme de deux carrés, peuvent se décomposer en somme de deux carrés de rationnels, d'une infinité de manières (II, 8 et 9).

2. Si deux entiers sont chacun la somme de deux carrés leur produit est la somme de deux carrés de deux manières (lemme III, 19).

Se plaçant ici dans l'optique de la théorie des nombres Abû Ja^cfar envisage dans l'ensemble des entiers naturels une série de jolies propositions (texte 35-41) dont certaines lui appartiennent probablement.

Texte 26 De $25 \pm 24 = \square$ nous tirons $\left(\frac{5}{2}\right)^2 \pm 6 = \square$. Comme $54:24 = \frac{9}{4}$ nous aurons $25 \cdot \frac{9}{4} \pm 54 = \square$ ou $\left(\frac{15}{2}\right)^2 \pm 54 = \square$.

Prenons $a = 720$, $720:2 = 360 = 40 \cdot 9$, avec $40^2 + 9^2 = 41^2$. Donc avec $41^2 \pm 720 = \square$, d'où par division par 9, $\left(\frac{41}{3}\right)^2 \pm 80 = \square$. D'autre
Texte 27 part $40:720 = 1:18$ qui n'est pas un rapport de carrés, donc on ne peut, par cette voie, trouver x rationnel tel que $x^2 \pm 40 = \square$.

Autrement. Si $a = 40$ il n'existe pas s et t (entiers) tels que $st = 20$
Texte 28 et $s^2 + t^2 = \square$. Pour savoir si $s^2 + t^2$ est un carré quand on a $st = a$ $(s < t)$ on divise s^2 par $2t$. Si le reste de la division égale le carré du quotient alors $s^2 + t^2 = \square$. (En effet, $s^2 = 2tq + q^2$ d'où $s^2 + t^2 = (t+q)^2$) Exemple: $a = 120 = 8 \cdot 15$, $360 = 9 \cdot 40$ etc. . . .

Si on n'a pas $s^2 = 2tq + q^2$ alors la recherche de x^2 tel que $x^2 \pm a = \square$ devient difficile ou impossible.

Il existe d'autres méthodes qui se ramènent toutes à la règle du
Texte 29 nombre 25. [$(3^2 + (2^2)^2 = 5^2$ soit la forme $x^2 + (y^2)^2 = z^2$)]. Par exemple, si nous trouvons (x, y, z) tel que $x^2 + (y^2)^2 = z^2$ nous prenons le rationnel $\frac{z}{y}$. Alors $\left(\frac{z}{y}\right)^2 \pm 2x = \frac{z^2 \pm 2xy^2}{y^2} = \frac{x^2 + (y^2)^2 \pm 2xy^2}{y^2}$ = carré de rationnel.

Ex.: $3^2 + (2^2)^2 = 5^2$ donne $\left(\frac{5}{2}\right)^2 \pm 2 \cdot 3 = \square$.

Ex.: $12^2 + (3^2)^2 = 15^2$ donne $\left(\frac{15}{3}\right)^2 \pm 2 \cdot 12 = \square$.

Pour trouver (x, y, z) tel que $x^2 + (y^2)^2 = z^2$, nous pouvons considé-
Texte 30 rer t^2 et $\frac{1}{4} t^2$ et poser $x^2 = (t^2 - \frac{1}{4} t^2)^2$, $(y^2)^2 = 4 \cdot t^2 \cdot \frac{1}{4} t^2$, $z^2 = (t^2 + \frac{1}{4} t^2)^2$. Nous aurons $\left(\frac{z}{t}\right)^2 \pm \frac{3}{2} t^2 = \square$, $\left(\frac{5t}{4}\right)^2 \pm \frac{3}{2} t^2 = \square$, $\frac{25t^2 \pm 24t^2}{4} = \square$.

Ex.: $t^2 = 16$, $x^2 = 12^2$, $(y^2)^2 = 256$, $z^2 = 400 = 20^2$,

$\frac{z}{t} = \frac{20}{4} = 5$, $\frac{3t^2}{2} = 24$, $5^2 \pm 24 = \square$.

Texte 31 Si l'on a un entier a tel que $\frac{2}{3} a = b^2$, carré d'entier, pour trouver

Le problème est impossible si a ne décompose pas en facteurs s et t conjugués.

Observation: Comme il a été dit, ce problème tient une place importante dans les recherches du 4ème siècle H · Il apparait en particulier dans les mémoires M2 et anonyme évoqués dans l'introduction. Dans les toutes premières années du 5ème siècle, al-Karaji par une méthode qui n'exclut pas le tâtonnement résout $x^2 \pm 5 = \square$ La même question ou des questions analogues se retrouvent dans les siècles postérieurs Ibn al-Hā'im reproduit, en les séparant, les équations $x^2 + 5 = \square$, $x^2 - 5 = \square$ (*Al-Ma'una*, écrit en 791 h., Ms Berlin 5984, pp. 290, 291) questions que l'on trouve antérieurement dans *al-Fakhri* d'al-Karajī (Ms Le Caire V, 212, f. 36ᵃ, l. 13, f. 59ᵇ, l 2). Ibn al-Khawām dans *al-Fawā'id al-bahā'iyya* (675 H.) cite $x^2 \pm 10 = \square$ parmi les 33 questions impossibles, "non, dit-il, que je prétende établir leur impossibilité, mais je déclare mon incapacité à les résoudre" (Ms. British Mus., Or 5615, f 44ᵃ). En Europe, la question $x^2 \pm 5 = \square$ est étudiée vers 1220 C. par Leonard de Pise, dans *Flos*, lequel aboutit par un chemin différent à la même réponse qu'al-Karajī $x^2 = \frac{1681}{144} = 11\,\frac{2}{3}\,\frac{1}{144}$

$$x = 3 + \frac{1}{4} + \frac{1}{6} = \frac{41}{12} \cdot$$

L'intérêt de $x^2 \pm a = \square$, comme l'a déjà relevé Woepcke, est qu'il est lié à des questions difficiles et fondamentales de l'analyse indéterminée qui ont été traitées par Fermat, Euler, Lagrange et Legendre (*Atti dell' Accademia Pont. N. Lincei*, "Recherches sur plusieurs ouvrages de Leonard de Pise", p. 252). On trouvera une ample documentation et des résultats intéressants sur la question dans L. E. Dickson, *History of the Theory of Numbers* (New York, 1952), vol 2, pp. 459-472. Relevons quelques énoncés · Genocchi a démontré en 1882 (ce qu'il avait énoncé en 1874) que $x^2 \pm a$ ne peuvent être tous deux des carrés de rationnels si a est premier de la forme $8k+3$ ou le produit de deux nombres premiers de cette forme, si a est le double d'un nombre premier de la forme $8k+5$ ou le double du produit de deux nombres premiers de cette forme. (*Dickson, op. cit.*, pp. 470, 467). Collins prouva en 1858 que pour $a < 20$, 5, 6, 7, 13, 14, 15 sont les seules valeurs de a pour lesquelles le système a des solutions (*Dickson, op. cit.*, p. 465). Destournelles prouva en 1881 l'impossibilité en nombres entiers du système $x^2 + y^2 = z^2$, $x^2 - y^2 = u^2$ (*Dickson, op. cit.*, p. 467).

Dans les mémoires M2 et anonyme déjà cités le problème était résolu au moyen de tables numériques et donnait lieu d'ailleurs à des remarques intéressantes. Utilisant les formules $z = s^2 + t^2$, $x = s^2 - t^2$, $y = 2st$, pour former les triangles rectangles numériques $z^2 = x^2 + y^2$ on a $z^2 \pm 2xy = x^2 + y^2 \pm 2xy = (x \pm y)^2$. D'où les différentes valeurs possibles pour a et pour z telles que $z^2 \pm a = \square$ (1). Ici l'auteur s'engage dans une autre voie et il recherche une condition nécessaire que doit remplir a pour que (1) soit possible, savoir a doit être de la forme $4m(2n+1)$ d'une part, d'autre part sa moitié doit être le produit de deux facteurs dont la somme des carrés est un carré. Ce qui constitue un critère commode pour les nombres relativement petits.

Texte 25 Si a est divisible par un carré m^2, alors $x^2 \pm a = \square$ donne $\left(\frac{x}{m}\right)^2 \pm \frac{a}{m^2} = \square$, égalité de la forme $x^2 \pm a = \square$ où x est rationnel.

Ex.: $289 \pm 240 = \square$ donne $\left(\frac{17}{2}\right)^2 \pm 60 = \square$. De même $\frac{289}{16} \pm 15 = \square$ ou $\left(\frac{17}{4}\right)^2 \pm 15 = \square$.

Texte 21 *Proposition: Si un nombre pair est la somme de deux carrés $a = b^2 + c^2$, sa moitié est une somme de deux carrés*, et la moitié de sa moitié aussi, et ainsi de suite tant que la moitié obtenue est un nombre pair.

$$\text{Car} \quad \frac{a}{2} = \frac{b^2 + c^2}{2} = \left(\frac{b+c}{2}\right)^2 + \left(\frac{b-c}{2}\right)^2.$$

Observation: L'auteur se rend bien compte que l'égalité est valable même pour des nombres fractionnaires. Les élégantes propositions 20,21 vont trouver leur application immédiate dans le problème suivant.

Problème: a est un entier donné. Trouver x tel que $x^2 \pm a = \square$. (1)

Supposons, par analyse, l'existence de x, y, z tels que $x^2 + a = z^2$ (2),
Texte 22 $x^2 - a = y^2$ (3). Evidemment $y < x < z$. Je dis que x^2 est une somme de deux carrés, car par addition $2x^2 = y^2 + z^2$, (4) donc x^2 est une somme de deux carrés (proposition précédente) (z et y ont même parité d'après (2) et (3)) et $x^2 = \left(\frac{z-y}{2}\right)^2 + \left(\frac{z+y}{2}\right)^2$. (5)

Par soustraction de (2) et (3) on a :

$$2a = z^2 - y^2 \qquad a = 2 \cdot \frac{z-y}{2} \cdot \frac{z+y}{2} \qquad (6)$$

Il en résulte que *a doit être pair*. Sa moitié $\frac{a}{2}$ est le produit de deux facteurs $\frac{z-y}{2}, \frac{z+y}{2}$ *qui ne peuvent être tous deux impairs ni tous deux de la forme* 2^n sans quoi (5) ne serait pas satisfait (lemmes 1 et 2).
Texte 23 $\frac{z-y}{2}$ et $\frac{z+y}{2}$ sont ou pairs tous deux ou l'un pair et l'autre impair. Dans tous les cas a est de la forme $a = 4m(2n+1)$. *Si cette condition n'est pas réalisée le problème est impossible.*

Texte 24 *Problème (suite): On donne a de la forme $4m(2n+1)$. Calculer x tel que* $x^2 \pm a = \square$.

Nous prenons les diviseurs s et t de $\frac{a}{2}$ tels que $\frac{a}{2} = st$, $s^2 + t^2 = \square$ s'il y en a.

Les nombres s et t sont alors dit *conjugués (qarināṇ)* $x^2 = s^2 + t^2$ *car* $s^2 + t^2 \pm 2st = \square$.

Le plus petit nombre a qui réponde à la question est $a = 24$, $\frac{a}{2} = 4 \cdot 3$, $3^2 + 4^2 = \square$. Puis parmi les multiples de 24 vient 240, $120 = 8 \cdot 15$, $8^2 + 15^2 = 17^2$, $x^2 = 17^2$, $17^2 \pm 240 = \square$.

(Cependant le mauvais choix de l'exemple numérique rend la règle difficile à saisir dans le texte).

Autre triplet (x, y, t). (C'est la première méthode particularisée)

Texte 16

$$x^2 = \left|\frac{a^4}{4} - a^4\right|^2 + 4\,a^4\cdot\frac{a^4}{4} = \left(a^4 + \frac{a^4}{4}\right)^2$$

ou $$\left(\frac{3a^4}{4}\right)^2 + (a^4)^2 = \left(\frac{5a^4}{4}\right)^2 \quad \text{Prendre } a^4 = 16$$

4ème méthode:

Texte 17 Prenons $p = s^2+t^2$ et a un entier tel que $a : s^2 - t^2$ égale un rapport de 2 carrés. $a(s^2 - t^2)$ est donc un carré [inclus dans la démonstration d'Euclide IX, 2; ou réciproque de VIII, 26 ajoutée par Héron et rapportée par al-Nairīzī (voir Heath, *The Thirteen Books*, vol. 2, p. 383)].

Donc $(ap)^2 = (as^2 + at^2)^2 = (as^2 - at^2)^2 + 4\cdot as^2\cdot at^2$.

On posera $y = as^2 - at^2$, $x = 2ast$ et $z = as^2 + at^2$.

L'exemple cité par l'auteur est $5 = 2^2 + 1^2$, $12 : 2^2 - 1^2 = 2^2 : 1^2$.
(Là encore de mauvais choix de l'exemple numérique rend la règle difficile à dégager).

Texte 18 Prenons deux carrés a^2 et b^2 tels que b^2 soit divisible par 4 et a^2b^2 bicarré. Ex.: 9 et $144 = (6^2)^2$.

$$\left|a^2 - \frac{b^2}{4}\right|^2 + 4a^2\,\frac{b^2}{4} = \left(a^2 + \frac{b^2}{4}\right)^2.$$

Texte 19 Autrement: Soient a et b deux nombres tels que ab soit un bicarré. Ex: 8 et 32, $32\cdot 8 = (16)^2$. Posons

$$\left(\frac{a}{4} + b\right)^2 = \left|\frac{a}{4} - b\right|^2 + 4\cdot\frac{a}{4}\cdot b$$

$$(2 + 32)^2 = (32 - 2)^2 + 4\cdot 2\cdot 32.$$

Cependant, dit l'auteur, cette méthode ne présente pas la régularité de celle décrite précédemment. (Peut-être entend-il que le choix de a et b n'obéit pas à une loi simple comme c'est le cas quand on opère sur des carrés a^2 et b^2) (texte 18).

Texte 20 *Proposition: Si un nombre a est une somme de deux carrés $a = b^2 + c^2$, son double est une somme de deux carrés.*

Car $2a = 2\,(b^2 + c^2) = (b - c)^2 + (b + c)^2$.

Par suite 2^2a, 2^3a, ... se divisent en somme de deux carrés.

Si x et y ont pour *p.g.c.d* d, par division par d^2 l'équation (1) sera amenée à la forme $x'^2 + y'^2 = (cs'^2)^2$, où c sera pris sans facteur carré.

Posant $cs'^2 = Z$ nous aurons $x'^2 + y'^2 = Z^2$ d'où $Z = M^2 + N^2$ comme plus haut. $M^2 + N^2 = cs'^2$ dépasse tout à fait les moyens de l'époque. Elle admet des solutions s'il existe un entier A tel que c divise $A^2 + 1$. Il en résulte alors que c est une somme de deux carrés. $c = f^2 + g^2$. Sa solution en est:

$$M = |(cq^2 - p^2)f \quad , \quad N = |p^2g - 2cpq + cgq^2 \quad . \quad s' = p^2 - 2gpq - cq^2|$$

Voir Dickson, *op.cit.*, p. 405 fin; Legendre, *Théorie des Nombres* (Paris, réimp. 1955), Tome I, p. 47 fin, Tome II, p. 203.

L'auteur appelle $(a^2 - b^2)^2$ et $4a^2b^2$ respectivement: petit et grand nombre $(a>b)$. En fait on peut avoir $(a^2 - b^2)^2 > 4a^2b^2$ ou $a^2 - b^2 > 2ab$, $a^2 - 2ba - b^2 > 0$, $(a-b)^2 - 2b^2 > 0$, $(a - b + b\sqrt{2})(a - b - b\sqrt{2}) > 0$, et comme $a > b$ il reste $a > b + b\sqrt{2}$ et $a > b(1 + \sqrt{2})$

Résolution de l'équation: $x^2 + (y^2)^2 = z^2$ (2)

Observation préliminaire: L'égalité $(a+b)^2 = (a-b)^2 + 4ab$ (3) montre que si on prend $(a-b)$ ou ab égal à un carré l'équation (2) sera satisfaite. De même, si l'on part de $a^2 + b^2 = c^2$, en multipliant les deux membres par a^2 ou b^2 on satisfait à l'équation (2). Les diverses méthodes de l'auteur se ramènent à des transformations de ce genre.

La solution générale de l'équation (2) s'obtient en posant $y^2 = Y$ et appelle les mêmes remarques que $x^2 + y^2 = z^2$. Cependant l'auteur dans les paragraphes 13-19 est sous l'influence de Diophante: au lieu de rechercher une solution générale dont il était capable, il multiplie les artifices en vue de recueillir un grand nombre de solutions particulières. La question sur laquelle convergent tous ses efforts est la résolution en nombres entiers et en nombres rationnels (rapports d'entiers) du système $x^2 \pm a = \square$ où a est un nombre donné, question qui tient une grande place dans les recherches du 4ème siècle H.

1ère *méthode pour résoudre* $x^2 + (y^2)^2 = z^2$ (2)

Texte 14 Prendre $x^2 = \left|\frac{b^4}{4} - a^4\right|^2$ et $(y^2)^2 = 4 \cdot a^4 \cdot \frac{b^4}{4}$. On aura alors $\left|\frac{b^4}{4} - a^4\right|^2 + 4 \cdot a^4 \cdot \frac{b^4}{4} = \left(\frac{b^4}{4} + a^4\right)^2$. On peut choisir $a^4 = 1$, $b^4 = 16$ et on aurait le plus petit triplet (x, y, z) vérifiant (2) $(4-1)^2 + 4 \cdot 1 \cdot 4 = (4+1)^2$.

2ème *méthode*:

Texte 15 L'égalité $4ab + (a-b)^2 = (a+b)^2$ montre que l'on peut choisir a et b tels que:

1) $a = ks^2$ $b = kt^2$ alors $4ab = (2kst)^2$,
2) $a - b = c^2$

12 et 3 sont des exemples de tels nombres a et b:

$$12 - 3 = 3^2 \quad 12 = 3 \cdot 2^2 \quad 3 = 3 \cdot 1^2$$

d'où $$4 \cdot 3 \cdot 12 + (12-3)^2 = (3 \cdot 4 + 3 \cdot 1)^2.$$

Recherche de 2, 3, 4 ... nombres dont la somme des carrés est un carré.

Texte 11 Nous pouvons trouver 2, 3, 4 . . . nombres dont la somme des carrés est un carré.

Cas de deux nombres. Prenons a^2 et b^2 quelconques, a^2b^2 et $\left(\frac{a^2-b^2}{2}\right)^2$ ont pour somme $\left(\frac{a^2+b^2}{2}\right)^2$: Démonstration par les segments. Cette égalité vaut pour des nombres fractionnaires. Mais dans ce dernier cas, nous ne dirons pas carré, mais *māl, à la manière des algébristes.*

Cas de trois nombres. Prenons $a^2 > b^2 + c^2$. Nous avons
$a^2b^2 + a^2c^2 + \left(\frac{a^2 - b^2 - c^2}{2}\right)^2 = \left(\frac{a^2 + b^2 + c^2}{2}\right)^2$.

Texte 12 Démonstration par les segments. Par ce procédé nous pouvons obtenir un grand nombre de triplets de carrés dont la somme est un carré.

Observation: Le procédé est généralisable et l'auteur s'en rend compte. Il n'explicite pas cependant l'égalité suivante. Si $a^2 > b^2 + c^2 + \dots k^2 + l^2$, alors:

$$a^2b^2 + a^2c^2 + \dots + a^2l^2 + \left(\frac{a^2 - b^2 - c^2 - \dots - k^2 - l^2}{2}\right)^2 = \left(\frac{a^2 + b^2 + c^2 + \dots + l^2}{2}\right)^2.$$

Texte 13 Trouver un triplet (x, y, z) tel que $x^2 + y^2 = (z^2)^2$. (1) Prendre un triplet (a, b, c) tel que $a^2+b^2 = c^2$. Poser $x^2 = (a^2 - b^2)^2$, $y^2 = 4a^2b^2$, d'où $x^2+y^2 = (a^2+b^2) = (c^2)^2$, $z = c$.

Observation: La solution donnée par al-Khāzin est partielle bien qu'ingénieuse. Nous pensons que l'auteur avait les moyens de résoudre

$$x^2 + y^2 = (z^2)^2 \qquad (1)$$

pour x, y, z premiers entre eux.

Posons $z^2 = Z$ d'où

$$x^2 + y^2 = Z^2 \qquad (2)$$

Comme x et y sont premiers entre eux, donc premiers avec Z, alors:

$$x = M^2 - N^2, \quad y = 2MN, \quad Z = M^2 + N^2$$

(M et N premiers entre eux et de parités différentes).

Par suite: $z^2 = M^2 + N^2$ a pour solution

$$M = m^2 - n^2 \quad , \quad N = 2mn \quad , \quad z = m^2 + n^2$$

(m et n premiers entre eux et de parités différentes).

Donc $x = (m^2 - n^2)^2 - (2mn)^2$, $y = 4mn\,(m^2 - n^2)$, $z = m^2 + n^2$.

D'ailleurs, quels que soient m et n, ces valeurs vérifient (1) car

$$[(m^2 - n^2)^2 - (2mn)^2]^2 + [4mn(m^2 - n^2)]^2 = (m^2 + n^2)^4$$

L'égalité (1) donne:

$$z = (ac + bd)^2 + (ad - bc)^2 \qquad (2)$$

$$z = (ac - bd)^2 + (ad + bc)^2 \qquad (3)$$

déjà rappelées et immédiates.

On obtient de la même manière:

$$z^2 = [(ac + bd)(ac - bd) + (ad - bc)(ad + bc)]^2 + [(ac + bd)(ad + bc) - (ad - bc)(ac - bd)]^2$$

Donc

$$z^2 = (a^2c^2 - b^2d^2 + a^2d^2 - b^2c^2)^2 + (a^2cd + abc^2 + abd^2 + b^2cd - a^2cd + abd^2 + abc^2 - b^2cd)^2$$

Puis $$z^2 = [(a^2 - b^2)(c^2 + d^2)]^2 + [2ab(c^2 + d^2)]^2$$

Ainsi on a bien obtenu la solution dérivée

$$(c^2+d^2)(a^2-b^2), \quad (c^2+d^2)(2ab), \quad (c^2+d^2)(a^2+b^2)$$

proportionelle à:

$$a^2 - b^2, \quad 2ab, \quad a^2 + b^2, \quad (a > b).$$

De même on verrait que

$$z^2 = [(ac+bd)(ad+bc) + (ad-bc)(ac-bd)]^2 + [(a^2c^2-b^2d^2) - (a^2d^2-b^2c^2)]^2$$

aboutit à $$z^2 = [(a^2+b^2)(2cd)]^2 + [(a^2+b^2)(c^2-d^2)]^2$$

solution proportionelle à

$$c^2 - d^2, \quad 2cd, \quad c^2 + d^2, \quad (c > d).$$

Texte Si x,y sont pairs, on a vu que $z = 2z' + x$, $z' + x$ nombre composé,
10 z' résidu *(fadla)*, $z' + x = s^2$, $z' = t^2$, $\sqrt{(z' + x)z'} = \frac{1}{2}y = st$ (y: *al-murabba' al-akthar*, x,y: *al-murabba*ᶜ*ayn al-awwalayn*) (texte 10, l. 4). D'où la conséquence que l'auteur énonce en général: quand un carré d'entier z^2 se décompose en une somme de deux carrés, sa racine z se décompose en une somme de deux carrés s^2 et t^2 qui sont premiers entre eux ou admettent un diviseur commun ou bien z se décompose en deux nombres plans semblables ($a \cdot b$ et $c \cdot d$ sont plans semblables si $a:b = c:d$, *Euclide* VII, déf. 21).

Observation: $x = s^2 - t^2$, $y = 2st$, $s > t$.

Si x et y sont premiers entre eux alors s^2 et t^2 sont premiers entre eux [si s^2 et t^2 ne sont pas premiers entre eux, s et t ont un diviseur commun d (conséquence d'Euclide VII, 27) et d diviserait x et y]. Plus généralement on peut avoir

1)	$x = k^2s^2 - k^2t^2$	$y = 2k^2st$	$z = k^2s^2 + k^2t^2$
ou 2)	$x = Ks^2 - Kt^2$	$y = 2Kst$	$z = Ks^2 + Kt^2$

avec K non carré dans 2). Dans ce dernier cas, Ks^2 et Kt^2 sont plans ensembles car $Ks.s$ et $Kt.t$ ont leurs côtés proportionnels $Ks:s = Kt:t$.

Cette égalité devient $s^2s'^2 + t^2t'^2 - t^2s'^2 - s^2t'^2 - 4ss'tt' = 0$ ou $(ss' - tt')^2 = (ts' + st')^2$.

En posant $ss' > tt'$, $ss' - tt' = ts' + st'$ qui donne $s'(s - t) = t'(s + t)$, $\frac{s'}{t'} = \frac{s+t}{s-t}$

Ainsi pour $s = 4$, $t = 3$, on a $s' = 7$, $t' = 1$.

D'où $\quad s^2 - t^2 = 7 \qquad 2st = 24 \qquad s^2 + t^2 = 25$

$$s'^2 - t'^2 = 48 \qquad 2s't' = 14 \qquad s'^2 + t'^2 = 50$$

On peut, par exemple, prendre s et t consécutifs.

Texte 9 Si on prend $s^2 = 4$ et $t^2 = 121$ *lesquels sont premiers entre eux* $z = 4+121 = 125$ est un multiple de 5, sans que $x = 121 - 4 = 117$ ni $y = 2 \cdot 2 \cdot 11 = 44$ ne soient équimultiples de 4 et 3. Comment expliquer la chose? [savoir que dans les triplets (x, y, z), (x', y', z') solutions, z soit multiple de z', sans que x et y soient des équimultiples de x et y']. Cela tient au fait que 125 est le produit de deux facteurs $(5 \cdot 25)$ qui se décomposent chacun en une somme de deux carrés $5 = 1+4$ et $25 = 9+16$. Tout nombre produit de deux facteurs qui sont chacun la somme de deux carrés se décompose en une somme de deux carrés, de deux manières, comme nous le verrons plus loin. $125 = 100+25 = 4+121$. D'où deux couples (s, t) différents pour un même z $125 = 10^2+5^2 = 11^2+2^2$. Quand z se décompose ainsi une des solutions (x, y, z) *n'est pas primitive*. Cela est comme le triangle primitif (3, 4, 5) qui donne naissance au triangle (dérivé) de côtés doubles (6, 8, 10).

Observation. Le couple (4,121) a fourni à l'auteur le triangle $117^2 + 44^2 = 125^2$ qui s'associe dans sa pensée avec $(25 \cdot 3)^2 + (25 \cdot 4)^2 = (25 \cdot 5)^2$. Al-Khāzin a l'air de se demander comment 125^2 s'est décomposé ainsi de deux manières différentes, et pourquoi la solution (75, 100, 125) n'est pas primitive? Cela tient, dit il, au fait que si deux nombres sont la somme de deux carrés, leur produit est une somme de deux carrés de deux manières

$$u = a^2 + b^2 \qquad v = c^2 + d^2$$

donne
$$uv = (ac + bd)^2 + (ad - bc)^2$$
$$uv = (ad + bc)^2 + (ac - bd)^2 \qquad \text{(Texte 39)}$$

Il montrera dans le texte 41 que si un nombre est une somme de deux carrés de deux manières, son carré est une somme de deux carrés de quatre manières (dont certains peuvent se confondre, c'est le cas pour 125^2).

$$125^2 = 120^2 + 35^2$$
$$125^2 = 100^2 + 75^2$$
$$125^2 = 117^2 + 44^2$$

L'idée d'al-Khāzin est difficile à suivre. Il semble partagé entre deux préoccupations: Partage d'un carré en somme de deux carrés de plusieurs manières, problème repris plus tard d'une façon si magistrale par Fermat [voir T. L. Heath, *Diophantus of Alexandria* (Cambridge Univ. Press, 1885; Dover repr.), pp. 106-110, 267-276] et la formation de triangles dérivés c-à-d., de la forme hx, hy, hz.

Montrons, en nous aidant des égalités employées par al-Khāzin, que si

$$z = (a^2 + b^2)(c^2 + d^2) \qquad (1)$$

alors, parmi les solutions de $x^2 + y^2 = z^2$, il y en a nécessairement qui sont dérivées.

Le système d'al-Khāzin est

$$\mathrm{II}\begin{cases} x = s^2 - t^2 & s > t \\ y = 2st \\ z = s^2 + t^2 \end{cases}$$

où *s* et *t* *sont premiers entre eux, l'un pair l'autre impair*. Il y a équivalence entre les deux systèmes. On voit en particulier en égalant les valeurs de *y* puis celle de *z*:

$$p^2 - q^2 = 4st, \qquad p^2 + q^2 = 2s^2 + 2t^2.$$

D'où
$$p^2 = (s+t)^2 \text{ et } p = s + t,$$
$$q^2 = (s-t)^2 \text{ et } q = s - t.$$

On a bien $x = pq = s^2 - t^2$.

Conclusion: Appelons triangle primitif *(aṣl)* ou solution primitive une solution (x, y, z) de nombres premiers entre eux. Celle-ci sera fournie par le couple (s,t) où s et t sont premiers entre eux et de parités différentes. L'idée sera reprise dans le paragraphe 8. Dans le paragraphe 6, al-Khāzin relève cependant que le système II, où s et t peuvent être quelconques, est toujours solution de $x^2 + y^2 = z^2$.

Texte 7 Pour $t^2 = 2^2$ et $s^2 = 3^2$, $x = 3^2 - 2^2 = 5$, $y = 2 \cdot 3 \cdot 2 = 12$, et $z = 3^2 + 2^2 = 13$. Le couple (5,12) est primitif *(aṣl)*. Il engendre des couples de nombres proportionnels dont la somme des carrés est un carré [c-à-d. $(5k)^2 + (12\,k)^2 = (13\,k)^2$] De même $(t^2, s^2) = (1^2, 4^2)$ donne $(x,y) = (15,8)$, $15^2 + 8^2 = 17^2$.

Texte 8 Ainsi pour former (x^2, y^2, z^2) on prendra (t^2, s^2) ***les plus petits carrés dans un certain rapport, ils sont donc premiers entre eux*** comme (1, 4), (4, 9), (1, 16) et on opérera comme plus haut. On n'obtiendra pas deux fois le même couple (x^2, y^2) ni deux couples proportionnels (l'expression arabe est vague: *ʿala ṣūratihimā*, à leur image).

Observation: En effet considérons deux couples générateurs (s,t), (s', t'). Il est facile de voir que si deux des trois rapports $\frac{s^2 - t^2}{s'^2 - t'^2}$, $\frac{2\,st}{2s't'}$, $\frac{s^2 + t^2}{s'^2 + t'^2}$, sont égaux alors $\frac{s}{t} = \frac{s'}{t'}$. Comme (s,t) et (s', t') sont des couples formés de deux nombres premiers entre eux alors $s = s'$, $t = t'$.

Ainsi dans le cas
$$\frac{s^2 - t^2}{s'^2 - t'^2} = \frac{st}{s't'}$$

$$s^2 s't' - t^2 s't' = sts'^2 - stt'^2$$

et
$$s^2 s't' - t^2 s't' - sts'^2 + stt'^2 = 0,$$

on a
$$ss'(st' - ts') + tt'(st' - ts') = 0,$$
$$(ss' + tt')(st' - s't) = 0,$$

d'où
$$st' = ts' \qquad \frac{s}{t} = \frac{s'}{t'}.$$

Les autres cas sont immédiats.

Cependant deux couples (s,t), (s',t') différents peuvent produire deux triplets (x,y,z) (x', y', z') tels que $\frac{x}{y'} = \frac{y}{x'} = \frac{z}{z'}$. Il suffit que $\frac{x}{y'} = \frac{y}{x'}$ ou $\frac{s^2 - t^2}{2\,s'\,t'} = \frac{2\,st}{s'^2 - t'^2}$.

Comme 12 et 3, $12:3 = 2^2 : 1^2$ d'où $12\cdot3$ et $12\cdot3\cdot4$ sont des carrés; de même 8 et 2. *Prenons $z'+x$ et z' les plus petits possibles* [donc premiers entre eux]. Nécessairement $z'+x$ et z' sont des carrés. [*Euclide*, VIII, 9].

Posons $z'+x = s^2$ et $z = t^2$. Dès lors $z = s^2+t^2$, $y = 2st$, $x = s^2-t^2$.

Texte 6 La règle qui donne (x, y, z) à partir de (s,t) est générale, [c-à-d. même si aucune condition n'est posée pour s,t les valeurs $s^2 - t^2$, $2st$, $s^2 + t^2$ vérifient $x^2 + y^2 = z^2$].

Pour $s = 2$, $t = 1$, $(x, y, z) = (3, 4, 5)$.

Pour $s = 3$, $t = 1$, $(x, y, z) = (8, 6, 10)$.

Remarquons que (8, 6, 10) sont doubles de (3, 4, 5). Plus généralement, si $x = 4k$, $y = 3k$, alors $z = 5k$.

Observation: Al-Khāzin utilise un langage visiblement influencé par Euclide quand il parle de x^2 impair et y^2 pair les plus petits possibles (texte 5, 1 2) [*Euclide* VII, 22, VIII, 2, 3, 4; IX, 15]. On trouve également chez Diophante. Etablissons donc maintenant deux triangles rectangles compris sous les moindres nombres, tels que 3, 4, 5 et 5, 12, 13 (*Arithmétique*, trad. Paul Ver Eecke Paris, 1959, livre III, 19, p. 109). Pour que l'expression d'al-Khāzin fût tout à fait claire, il eut fallu dire: les plus petits possibles dans leur rapport. Nous pensons que c'est la pensée d'al-Khāzin, car si on devait prendre à la lettre l'expression *les plus petits possibles* l'équation $x^2+y^2 = z^2$ n'aurait qu'une solution (3, 4, 5) alors que l'auteur en donne plusieurs dans le paragraphe même. La même expression utilisée plus loin à propos de $z'+x$ et z' dans $(z'+x):z'$ ne présente plus le même inconvénient puisque le rapport de $z'+x$ et z' est formé. L'expression correcte des deux plus petits nombres dans leur rapport est utilisée au début du texte 6. Pour la rigueur du raisonnement il nous resterait à établir que si x et y sont premiers entre eux (donc x et z le sont aussi) il en est de même de $z+x$ et z', et réciproquement, ce qui ne présente aucune difficulté.

Le texte ne précise pas que s et t doivent être de parités différentes (si s et t étaient de même parité $x = s^2-t^2$ et $z = s^2+t^2$ seraient pairs tous deux ce qui est contraire au texte).

Euclide a montré que

$$\mathrm{I}\quad \begin{cases} x = mnpy \\ y = \frac{1}{2}\,(mnp^2 - mnq^2) \\ z = \frac{1}{2}\,(mnp^2 + mnq^2) \end{cases}$$

est solution de $x^2 + y^2 = z^2$ (*Eléments* X, 29, lemme 1). Cependant Euclide ne donne que la synthèse et par là il manque d'établir *que la solution proposée est générale* Pour cette raison, Bachet en donne l'analyse dans son édition de Diophante (1621).[1] *C'est justement ce qu'al-Khazin a fait ici.*

Nous pouvons nous en tenir aux valeurs de (x, y, z) premières entre elles dans leur ensemble. Le système d'Euclide devient $x = pq$, $y = \frac{1}{2}\,(p^2 - q^2)$, $z = \frac{1}{2}\,(p^2+q^2)$, où p et q *sont premiers entre eux et impairs.*

1. Jean Itard, *Les livres arithmétiques d'Euclide*, (Paris 1961), p. 163.

Construction en entiers de $x^2 + y^2 = z^2$

Propositions préliminaires

Lemme 1. *Deux carrés impairs ne peuvent avoir pour somme un carré.*

Texte 2 Supposons que $x^2 + y^2 = z^2$, x et y étant *impairs*. Donc z est pair (*Euclide*, IX, 22). De plus

$$x^2 = (z-y)\ (z+y) = (z-y)^2 + 2y(z-y).$$

Mais $$[x-(z-y)]\ [x+(z-y)] + (z-y)^2 = x^2.$$

Il s'ensuit que $[x-(z-y)]\ [x+(z-y)] = 2y(z-y)$.

Les crochets sont pairs tous deux. Dans le 2ème membre y et $z-y$ sont impairs. Donc l'égalité est impossible.

Texte 3 *Lemme* 2. *Deux carrés de la forme* 2^n *ne peuvent avoir pour somme un carré.*

Si $x = 2^p$ et $y = 2^q$ (avec $p<q$) on ne peut avoir $x^2+y^2 = z^2$. Il existe s tel que $\frac{x}{y} = \frac{1}{2^s}$ [*Euclide* IX, 11; voir observation de T. L. Heath, *The Thirteen Books*, vol. 2, p. 396]. D'où $\frac{x^2}{y^2} = \frac{1}{(2^s)^2}$ et $\frac{x^2}{x^2+y^2} = \frac{1}{(2^s)^2+1}$.

Or $(2^s)^2+1$ n'est pas un carré, car en ajoutant 1 à un carré on n'obtient pas un carré. Par suite $x^2 + y^2$ ne peut être un carré [si $x^2 + y^2$ était un carré, alors $(2^s)^2+1$ serait un carré d'après *Euclide* VIII, 24].

Texte 4 *Lemme* 3. $(2m+2n+1)^2 = (2n+1)^2+4m(2n+1+m)$, [*Euclide* II, 8]

$$(2m+2n)^2 = (2m)^2+4(2m+n)n$$

Observation Les démonstrations dans les lemmes 1, 2, sont faites sur des segments comme dans les Eléments d'Euclide.

Formation de $x^2 + y^2 = z^2$

Nous voulons trouver deux nombres carrés l'un impair x^2 l'autre
Texte 5 pair y^2 [premiers entre eux] (dans le texte: les plus petits possibles) tels que $x^2+y^2 = z^2$. *Supposons par l'analyse qu'ils existent.* (Posons $z-x = 2z'$). Appelons $z'+x$: nombre composé (*ʿadad murakkab*) et z': résidu (*faḍla*). Alors $z = (z'+x)+z'$ et $x^2+4(z'+x)z' = z^2$ [lemme 3]. Mais $z^2 = x^2+y^2$ d'où $4(z'+x)z' = y^2$. Il en résulte que $(z'+x)z'$ est un carré, car le rapport de $4(z'+x)z'$ à $(z'+x)z'$ est le rapport d'un carré à un carré et $4(z'+x)z'$ est un carré, donc $(z'+x)z'$ est un carré (*Euclide*, VIII, 24). Par suite $\frac{z'+x}{z'}$ est un rapport de deux carrés (*Euclide* IX, 2, puis VIII, 26) et $z'+x$ et z' sont des équimultiples des plus petits carrés qui ont le même rapport qu'eux.

est confirmé par l'histoire.[14] Un autre traité sur les triangles rectangles du mathématicien Abū'l-Jūd, 2e moitié du 4e siècle. H., vient d'ailleurs étayer toutes les vues précédentes.[15] Signalons également sur le même sujet un traité d'al-Sijzī (2e moitié du 4e s. H.): *Risāla fī jawāb mas'ala ᶜadadiyya wa hiya kaifa najid (murabbaᶜyn yakūn) majmūᶜuhumā murabbaᶜa* (12 pages, Bibl. Ḥakim M. Nabī Khān Jamāl Suwayda, Téhéran). Nous devons à la courtoisie du Dr. Anton M. Heinen d'en avoir pris connaissance.

14. Woepcke, *op. cit.*, p. 317.
15. Leiden Cod. Or. 168 (14), f. 116-134a.

Sommaire du traité d'Abū Jaᶜfar [*al-Khāzin*].

Paris BN MS arabe 2457,49, *ff.* 204a - 215a.

Ce sommaire n'est pas à proprement parler une traduction, cependant nous croyons qu'il ne laisse rien échapper du texte. Les passages importants ou difficiles y ont reçu des développements plus grands. D'autre part, les démonstrations d'al-Khāzin bien qu'exposées sur des exemples numériques sont générales et entendues par l'auteur comme telles: nous n'exagérons donc pas leur portée en représentant les nombres par des lettres, ce qui a l'avantage de rendre les démonstrations plus claires. Des observations imprimées en petits caractères et précédées de la mention *observations* accompagnent certaines questions et sont étrangères au texte; de même en est-il des expressions placées entre crochets dans le texte même. Dans un souci de meilleure présentation et pour faciliter le travail de référence nous avons sectionné le mémoire en paragraphes.

Remarques

1. Nous avons mis en italique dans le texte certains mots ou phrases clés. Le nombre au dessous du mot *texte* désigne le numéro du paragraphe.

2. Nous employons le signe $\square$ pour désigner un carré d'entier (ou parfois de rationnel: rapport d'entiers).

3. Les nombres dont il est question – sauf mention expresse du contraire – sont des entiers naturels.

4. Certaines phrases insérées entre crochets n'appartiennent pas au texte et sont ajoutées en annotations.

ques mémoires qui nous sont restés sur $x^2 + y^2 = z^2$ nous font revivre les efforts conjugués, les erreurs commises, les insuffisances et les corrections successives. Nul doute qu'à cet effort collectif d'édification bien des mathématiciens célèbres ou obscurs n'aient participé dans les divers centres scientifiques: Baghdad, Chiraz, Rayy, Marw, Balkh, et autres.[12]

La préface de **M3** presente un détail historique qui confirme cette persistance dans l'effort. Motivant l'envoi de son mémoire, Abū Jaᶜfar écrit: Frère je t'avais adressé un mémoire sur la construction des triangles rectangles. J'y avais énoncé, sans démonstration par les segments, que deux nombres dont la somme des carrés est un carré ne pouvaient être impairs (on aura remarqué la ténuité du résultat). Or cette proposition est absente du mémoire **M2** et il est difficile de lui trouver là une place naturelle dans l'enchaînement du raisonnement. Il faut donc admettre qu'Abū Jaᶜfar fait allusion à un 3ᵉ mémoire qu'il avait adressé également à ᶜAbdallāh b. ᶜAlī. La chose n'a rien qui nous surprenne. Il est tout normal qu'Abū Jaᶜfar, et les autres chercheurs creusant la question, aient rédigé au fur et à mesure bon nombre de notes brèves sur ce sujet alors à l'ordre du jour.

Nous possédons d'ailleurs sur les triangles rectangles numériques un fragment de traité anonyme, Paris MS 2457, ff. 81a–86a, dont la qualité montre un progrès sensible sur le mémoire **M2** d'Abū Jaᶜfar. Les deux traités **M2** et anonyme, ne manquent pas d'ailleurs de points de ressemblance, ce qui avait fait dire à F. Woepcke, à une époque où les conditions de l'activité scientifique arabe étaient moins claires: "On ne pourra méconnaître l'uniformité que présente en général la marche suivie dans l'exposé de la théorie des triangles rectangles numériques, tant par l'auteur du fragment anonyme que par Abou Dja'far M. b. al-Hoçain, uniformité qui pouvait indiquer une certaine tradition d'école, un certain cadre commun qu'il était d'usage de remplir, en enrichissant d'ailleurs le sujet d'autant d'observations et de découvertes originales que possible.[13] F. Woepcke en venait à supposer qu'il existait des rapports plus ou moins suivis entre les mathématiciens d'Orient, ce qui

12. De cette multiplicité d'efforts, bien naturelle d'ailleurs, nous donne une idée le bref chapitre des triangles rectangles numériques $(a^2 = b^2 + c^2)$, (3), qu'al-Samaw'al insère dans son livre *al-Bahir* cité en note 1. al-Samaw'al y est représenté par $2(a-c)(a-b) = [a-(a-c)-(a-b)]^2$ conséquence de(3). Al-Sijzī par l'égalité bien connue et tres ancienne $a^2 \pm 2bc$ sont des carrés; Ibn al-Haytham par la construction d'un triangle rectangle dont un côté de l'angle droit est connu (*al-Bāhir*, *op. cit.*, pp 146-151). Dans un chapitre voisin, al-Samaw'al cite un nom obscur, Jaᶜfar b. ᶜAbdallāh al-Ḥarīrī (pp. 155, 159, 117) auteur de l'identité $b(a+b+c) + ac = (a+b)(b+c)$. D'autre part on doit à Ibn Yūnus une note sur la proposition, "Deux carrés impairs n'ont pas pour somme un carré", Berlin 6008, ff. 437a-438b.

13. F. Woepcke a traduit et analysé remarquablement les traités, Paris MS 2457, ff 81a-86a anonyme, et celui d'Abū Jaᶜfar, Paris MS 2457, ff 86b-92a, "Recherches sur plusieurs ouvrages de Leonard de Pise. . .," *Atti dell'Accademia Pontificia de Nuovi Lincei*, 14 (1861), pp. 211-227, 241-269 (pour le traité anonyme): pp. 301-324, 343-356 (pour le 2ᵉ traité), cf. p. 317

personnage qui a joué le rôle important d'intermédiaire et d'arbitre entre les savants de son temps et à qui sont adressés d'ailleurs les deux mémoires M2 et M3.[9] Cette discussion est intéressante car elle nous révèle l'existence d'une correspondance scientifique entre les mathématiciens – ce dont nous avons par ailleurs de nombreux témoignages[11] – ainsi que les tentatives répétées entreprises par les Arabes, tôt dans la première moitié du 4e siècle H., pour résoudre $x^2 + y^2 = z^2$ (1) ou la difficile $x^3 + y^3 = z^3$ (2). *Les* quel-

11 La correspondance joue un rôle important dans la vie scientifique de l'époque elle supplée les déficiences de l'édition et épargne aux consultants des voyages longs et pleins de risques, en même temps qu'elle assure aux consultés une plus grande notoriété et aussi des sujets de recherche Bien des écrits ont vu le jour sur une sollicitation amicale Dans l'Orient d'hier et de jadis où le temps n'avait pas valeur de monnaie ces demandes ne semblaient pas déplacées. Citons les 15 lettres adressées par Abū Naṣr b. ʿIrāq à son élève al-Bīrūnī pour lever certaines de ses difficultés mathématiques et où il l'encourage dans la voie de l'étude (Hayderabad, 1948), la réponse d'al-Sijzī à dix questions que lui avait adressées un géomètre de Chiraz, Paris MS 2457, 151a 136b, la lettre d'al-Sijzī (Aḥmad b. Muḥammad b. ʿAbd al-Jalīl) à Abu'l-Ḥusayn Muḥammad b. ʿAbd al Jalīl (son pere) et dont il dit être d'esclave, *min ʿabdih* (Paris MS 2457, 137b-139a). Ibn Ṭāwūs (m. 664 H.) nous apprend dans *Faraj al-mahmūm fī tarīkh ʿulamā' al-nujūm* (al-Najaf, 1368 H.), p. 127, que le père d'al-Sijzī, M b. ʿAbd al-Jalīl était versé dans la science des astres et qu'il était l'auteur de livres connus à l'époque d'Ibn Ṭāwūs *Kitāb al-zijāt fī istikhrāj al-hylāj w'al-kadkhudā* et *Maqala fī fath al-bāb* (l'édition très fautive porte al-Sinjarī au lieu d'al-Sijzī, erreur due au déplacement d'un point diacritique Citons aussi la lettre d'al-Sigzī à Abū ʿAlī Naẓīf b Yumn en 970 A.D MS Paris 2457 f 136b-137a, la lettre d'al-Hāshimī (vit en 320 H.) à l'émir Abū'l-Faḍl Jaʿfar b. al-Muktafī sur le calcul des radicaux, MS Paris 2457, 16, f 76a-78a, la correspondance entre Abū Jaʿfar al Khāzin et le geomètre Ibrahim b. Sinān (296-335 H 908 - 946 A.D.) qui commença sa carrière de chercheur à l'âge de 15 ans (Ibn ʿIrāq, *Rasā'il- Taṣḥīḥ zīj al-Ṣafā'iḥ* (Hyderabad, 1948), p. 45, Ibrahim b. Sinān, *Rasā'il- Kitāb fī ḥarakāt al-shams* (Hayderabad, 1948), p. 70 la correspondance entre al-Buzjānī (m 387 H.) et le cadi mathématicien Abū ʿAlī al-Ḥubūbī (Ibn ʿIrāq, *Rasā'il Al-qusiyy al-falakiyya* (Hyderabad, 1948), p. 2, l'abondante correspondance d'Abū'l-Jūd Ibn al-Layth avec ses contemporains Al Sijzī (Leiden Cod. Or 168, 13, 108b-113) avec al-Bīrūnī *op cit.*, f 45a-54a), avec Ibn al-Ghadī ? (*op cit.*, f 116-134a), avec Abū Jaʿfar al-Khāzin (*op cit.*, f 102-108a), voir aussi notre article "*Tasbīʿal-dā'ira*", *JHAS*, 1 (1977), 379 380, 373 Rappelons aussi la correspondance scientifique avec les pays musulmans de Frédéric II, (1194-1250 A D.) qui connaissait l'arabe et aussi le grec, le latin, l'italien, l'allemand et le français (Amari, "Questions philosophiques adressées aux savants musulmans par l'empereur Frédéric II", *Journ. As.*, 5e s., 1 (1853), 240-274, A F. Mehren, "Correspondance du philosophe soufi Ibn Sabʿin Abd oul-Haqq avec l'empereur Frédéric de Hohenstaufen sur l'immortalité de l'âme," *Journ As.*, 7e s., 14 (1879), 342-344, 347, Aldo Mieli, *La Science Arabe* (Leiden, 1966), pp 152, 209 G Sarton, *Introd.*, vol II, part II, p 600 et pp 575-579 Al-Qazwīnī, *Āthār al-bilād wa akhbār al-ʿibād* (Gottingen, 1848), p. 310 (Voir aussi Ibn Khallikān, *Wafayāt al-Aʿyān*, vol. 4, (Caire, 1948), pp. 396 et suiv., où un habitant de Damas intéressé par les mathématiques ecrit à Ibn Yūnus (Mossoul) et reçoit quelques mois plus tard la réponse à ses difficultés (en 633 H.) Arrêtant ici une énumeration que nous pourrions allonger considérablement disons la nécessité de la correspondance entre astronomes observant en des lieux differents pour concerter leurs observations et remarquons que dans de nombreux manuscrits les en-tête des mémoires ont disparu cachant ainsi le caractère epistolaire des écrits D'autre part cette pratique est commune à toutes les branches du savoir. Ansi Āqā Buzurg, dans sa Ḏarīʿa, vol 2, (Najaf, 1355 H.), pp. 71-94, donne une longue énumération de 186 traités religieux, juridiques ou philosophiques composés en réponse à des questions posées par des correspondants, et il considère que la plus grande partie des mémoires dûs à la correspondance a dû se perdre.

la théorie des nombres en général. Diophante y est nommé expressément.[8] Ce mémoire que nous désignerons sous le sigle **M3** traite de la résolution en nombres entiers de $x^2+y^2=z^2$, de $x^2+(y^2)^2=z^2$, $x^2+y^2=(z^2)^2$ et d'un certain problème que l'on peut qualifier de diophantien, encore qu'il ne figure pas absolument dans l'Arithmétique de Diophante. Calculer x rationnel pour que x^2+K égale un carré de rationnel. Il existe un 2e mémoire d'Abū Jaᶜfar **M2** sur le même sujet: construction des triangles rectangles en nombres entiers, Paris MS 2457 fol. 86b-92b mais la méthode d'approche de la solution y est tout à fait différente.

L'auteur y construit un tableau numérique donnant tous les triplets (x, y, z) solutions de $x^2+y^2=z^2$ jusqu'à $z \leqslant 461$ et y étudie diverses propriétés de ces triangles.[9] Ce mémoire est apparemment antérieur à **M3** si on en juge par les inadvertances et les erreurs qui s'y rencontrent.[10] On sent que l'auteur n'a pas acquis la pleine maîtrise de son sujet alors que dans **M3** la solution de $x^2+y^2=z^2$ se présente sous une forme élégante, presque classique, comme on le verra. La préface de **M2** est intéressante du point de vue historique, elle nous apprend qu'Abū Jaᶜfar avait été précédé dans sa tentative par Abū Muhammad al Khujandī, mais que la formule établie par ce dernier pour la solution de $x^2+y^2=z^2$ n'était pas générale. De même Abū M. al-Khujandī avait cru démontrer l'impossibilité de $x^3+y^3=z^3$ en nombres entiers, mais Abū Jaᶜfar avait montré son erreur. Il en avait résulté une discussion entre les deux auteurs, discussion qu'avait suivie ᶜAbdallāh b. ᶜAlī l'arithméticien.

8. MS Paris 2457, ff. 213a, 214b.

9. Si x, y, z n'ont pas de diviseur commun alors la solution générale de l'équation $x^2+y^2=z^2$ est $z=a^2+b^2$, $y=a^2-b^2$, $x=2ab$, où a et b sont premiers entre eux, l'un pair, l'autre impair. Par suite pour obtenir toutes les valeurs possibles de z, Abū Jaᶜfar écrit dans une 1ère colonne, les nombres 1, 2, 3, ..., n; dans une 2e colonne leurs carrés $1^2, 2^2, 3^2, \ldots, n^2$. Il ajoute alors 1^2 à $1^2, 2^2, \ldots, n^2$ et écrit les sommes obtenues dans la ligne horizontale passant par 1. Puis il ajoute 2^2 à $2^2, 3^2, \ldots, n^2$, et écrit les résultats dans la ligne horizontale passant par 2. Il suffit de choisir dans les lignes horizontales les z impairs: a^2 et b^2 en découlent d'où y et x.

1	1	2	5	10	17	26	37
2	4	8	13	20	29	40	
3	9	18	25	34	45		
4	16	32	41	52			
5	25	50	61				
6	36	72					

Ainsi $17 = 1 + 16 = 1^2 + 4^2$.
Par suite $y = 4^2 - 1^2 = 15$ et $x = 2 \cdot 4 \cdot 1 = 8$.
$29 = 4 + 25 = 2^2 + 5^2$
Donc $y = 5^2 - 2^2 = 21$ et $x = 2 \cdot 5 \cdot 2 = 20$.

10. Les étourderies ou les erreurs sont fréquentes, semble-t-il, dans l'œuvre d'al-Khāzin. Le mémoire **M3** n'en manque pas; et voir: Abū Naṣr b ᶜIrāq, *Taṣḥīḥ ṣīḥ al-ṣafā'iḥ* (*Rasā'il Abī Naṣr*, Hyderabad, 1948), Al-Bīrūnī, *Tamhīd al-mustaqirr*, (*Rasā'il al-Bīrūnī*, Hyderabad, 1948), pp. 77-78.

en même temps qu'il traduit l'Arithmétique de Nicomaque et revoit la traduction des Eléments d'Euclide[2] fait des propriétés des nombres l'objet de ses méditations et on lui doit des écrits qui restent parmi les œuvres mathématiques arabes les plus profondes en même temps qu'il frôle le raisonnement récurrentiel dans certaines relations numériques.[3] A en juger par la liste de ses ouvrages, il ne semble pas que Thābit se soit intéressé à l'Arithmétique de Diophante. De ce livre aucune trace non plus dans l'Algèbre pourtant si riche de Shujā^c b. Aslam, qui d'après nous a fleuri autour de 265 H.[4]

Dès le début du 4e siècle H. l'influence de Diophante se fait cependant sentir et elle persistera jusqu'à la fin du siècle et bien entendu au-delà. Al-Būzjānī (m. 387 H.), venu de la Perse Orientale touche Baghdad en 348 H.,[5] à un moment où Baghdad vit des années relativement calmes sous le règne du bouyide Mu^cizz al-dawla.[6] Il écrit un "Commentaire sur le livre de Diophante" un "livre d'initiation à l'Arithmétique" (théorie des nombres ou livre de Nicomaque ?), le "livre des démonstrations employées par Diophante et celles employées par l'auteur dans son Commentaire".[5] Or, avant d'arriver à Baghdad il avait reçu son instruction sur la théorie des nombres, *al-^cadadiyyat*, et les questions arithmétiques de ses oncles Abū ^cAmr al-Maghazilī et Abū ^cAbdallāh M. b. ^cAnbasa, auteurs d'ouvrages perdus.[5]

Le mémoire que nous publions; Paris MS 2457, f. 204a – 215a, appartient à un auteur qui est également de la Perse Orientale: Abū Ja^cfar Muḥammad b. al-Ḥusayn al-Khurāsānī al-Ṣāghānī al-Khāzin dont le nom et l'activité remplissent la première moitié du 4e siècle H.[7]

Objet du mémoire

Le mémoire d'Abū Ja^cfar relève de cette catégorie d'ouvrages nés sous le signe de l'activité qui règne autour de l'Arithmétique de Diophante et de

La formule attribuée par Proclus à Platon pour la construction des triangles rectangles numériques était connue des Arabes, $[(m-1)(m+1)]^2 + (2m)^2 = (m^2+1)^2$. Elle figure, par ex., dans un mémoire anonyme dont il sera question plus tard (voir note 14).

2. *Al-Fihrist*, p. 385. Al-Qifṭī, *Ikhbār*, p.47; T. L. Heath, *The Thirteen Books of Euclid's Elements* (New York, Dover Publ., 1956), vol. 1, pp. 75-76.

3. F. Woepcke, "Notice sur une théorie ajoutée par Thābit b. Korrah", *Journ. As.*, 20 (1852), 4e s., 420-429 (sur les nombres amiables). Voir le jugement de G. Sarton sur les quadratures de Thābit, *Introd.*, vol. 1, p. 600.

4. Ibn Aslam, *Al-Jabr w'al-muqābala*, MS Qara Mustafa, 379. Adel Anbouba, *Un algébriste arabe*, (Beyrouth, 1963), Horizons Techniques du Moyen Orient, no 2, pp. 6-15. Adel Anbouba, "L'algèbre arabe aux IXe et Xe siècles. Aperçu général," *Journal for the History of Arabic Science*, 2(1978), 66-100.

5. *Al-Fihrist*, p. 408; al-Qifṭī, p. 166.

6. Voir les évènements des années 336-350 H., dans Ibn al-Jawzī, *Al-Muntaẓim* (Hyderabad, 1357-8H.), vols. 6 et 7.

7. Pour quelques détails biographiques sur Abū Ja^cfar (et ^cAbdallāh b. ^cAlī dont il sera question plus loin) on voudra bien se reporter à l'article Anbouba, "L'algèbre arabe", pp. 89-90. Voir aussi pp. 98-100.

Un Traité d'Abū Ja'far [al-Khazin] sur les triangles rectangles numériques

Adel Anbouba*

Introduction

L'intérêt des Arabes pour la théorie des nombres a commencé aussitôt que le 3e siècle H. A la base de cet intérêt se placent les trois livres arithmétiques des Eléments d'Euclide, le Xème, l'Arithmétique de Nicomaque de Gérase, l'arithmétique de Diophante, certaines questions de quadratures et à n'en pas douter des traités ou fragments de traités grecs obscurs qui ne nous sont pas parvenus, voir même des passages de philosophes grecs.[1] Thābit b. Qurra

* Institut Moderne du Liban, Fanar-Jdaidet, Beyrouth, Liban. Cet article envoyé à l'edition aussi tôt que mai 1978 a subi, comme on le voit, un retard accidental assez long. Entre temps nous avons appris que le Dr. Ahmad Saidan avait publié dans la revue *Dirāsāt*, de l'Université Jordanienne, (décembre 1978), le mémoire objet de notre article (avec une analyse en langue anglaise): Paris MS 2457, 49 (non 41), ff 204a- 215a. Nous nous sommes demandé alors si nous ne renoncerions pas à notre publication Mais outre qu'une variété d'éditions d'un même texte ancien peut être de quelque utilité pour les chercheurs, nous avons pensé que la partie française de notre article en justifiait l'apparition. Il est vrai que le Dr. Saidan écrit "This is the text of the tract translated by Woepcke in *Atti dell' Acc pontif d. nuovi Lincei* 14 (1861). It is edited to form chapter two. . ." (*op.cit.* p.7). En fait, Woepcke dont la vie fut, hélas, assez brève, n'a pas traduit en français le texte concerné ici, mais Paris MS 2457, 19, ff 82-86a, fragment d'un traité anonyme et MS 2457, 20, ff. 86b-92a d'Abū Ja'far dont on trouvera l'analyse française dans Woepcke, op.cit. pp. 211-227, 241-269, et pp 301-324, 343-356 respectivement. Nous profitons de cette occasion pour remercier ici le Conservateur des manuscrits orientaux à la Bibliothèque Nationale de Paris, Mlle M.-R Séguy dont nous avions sollicité et obtenu, au début de 1978, l'autorisation de publier le mémoire d'Abū Ja'far. Notre reconnaissance va également à Mlle M.-T. Debarnot qui a lu avec beaucoup de soin le sommaire français de notre article et dont les remarques et les suggestions nous ont permis de reprendre la rédaction de certains passages et d'y apporter des rectifications.

1 Nous ignorons si des commentaires de l'Arithmétique de Nicomaque furent traduits en arabe; la chose est plausible, les noms des commentateurs Proclus, Jean Philopon, Jamblique n'étaient pas étrangers aux Arabes. (Al-Qifṭī, *Ikhbār al-ʿulamā'* (Caire, 1326 H.), pp. 44, 70, 232. G. Sarton, *Introduction to the History of Science* (Baltimore, 1927) vol. 1, pp. 253, 351 T. L. Heath, *A Manual of Greek Mathematics* (Oxford, 1931), p. 62 Al-Qifṭī cite de Proclus d'Alexandrie un ouvrage sur "la nature des nombres, en 6 livres" (Dans l'édition, Proctus pour Proclus). Ibn al-Nadim nous apprend qu'on avait écrit des abrégés du livre de Nicomaque (*al-Fihrist*, Caire, s. d. p. 391). On doit à al-Kindī (m. 257 H.) un mémoire sur les nombres employés par Platon dans sa *Politique* (*al-Fihrist*, p. 373). Al-Samaw'al 6e s. H cite un livre sur les nombres, apparemment apocryphe, attribué à Pythagore. (*Al-Bāhir*, éd. Ahmad et Rashed, (Damas, 1972), pp 9, 120, 122. Que l'on compare les nombres évoqués comme bases de numération par al-Jāḥiẓ (*al-Tarbīʿ w'al-tadwīr*, éd. Pellat, (Damas, 1955), p.81) et le nombre choisi par Platon pour la population de la cité idéale (*Les Lois*, III, VI, coll. des U. de France, Tome XI, 2e p., trad. E. des Places, (Paris, 1951), p. 92).

ملخصات الأبحاث المنشورة في القسم الأجنبي

المصدر الأصيل هيئة الكواكب المنسوبة الى قطب الدين الشيرازي

جورج صليبا

لقد نُشرت قبل اثني عشرة سنة دراسة* وصفت فيها هيئة الكواكب العليا كما ارتآها قطب الدين الشيرازي . وفي تلك الدراسة اثيرت بعض الشكوك حول تلك الهيئة وحول كونها من ابتكار قطب الدين نفسه ام انه اخذها عن فلكي سابق له .

في هذا المقال نثبت نصاً من مخطوط محفوظ في اكسفورد تحت رقم مارش ٦٢١ نبرهن فيه ان الهيئة المنسوبة الى قطب الدين الشيرازي كانت في الواقع من تأليف الشيخ الامام مؤيد الدين العرضي الذي صنف هيئته تلك قبل مجيئه الى مراغه وقبل ان يؤلف قطب الدين هيئته بحوالي ثلاثين سنة تقريبا . ولما كانت الهيئتان متطابقتان كان لا بد من اعتبار هيئة قطب الدين نسخة عن الهيئة التي ابتكرها مؤيد الدين العرضي .

والنص الذي يثبت عدم اصالة هيئة قطب الدين هو ما قاله هو بنفسه في كتابه « نهاية الادراك » والذي الفه سنة ١٢٨١ م ، حين قال :

« قال بعض افاضل المتأخرين من اهل الصناعة ههنا ان الشيء الذي يجعل علامة لمبدأ حركة يجب ان يكون ساكناً بالنسبة الى المتحرك ليكون تباعد المتحرك عنه وتقاربه اليه بحركة المتحرك وحده »

فلا يمكن ان يكون قطب الدين يتكلم عن نفسه عندما يذكر « بعض افاضل المتأخرين » وعندما ينسب الى هذا المجهول رأياً لا يوافقه عليه . اما المجهول هذا فليس سوى مؤلف المخطوط مارش ٦٢١ وهو مؤيد الدين العرضي المتوفي سنة ١٢٦٦ م اذ يقول :

« ان الشيء الذي يفرض علامة لمبدأ حركة متحرك يجب ان يكون ساكناً بالنسبة الى المتحرك ليكون تباعد المتحرك عنه وتقربه اليه انما هو بحركة المتحرك وحده » (مارش ٦٢١ ص ١٢٤ ظ) .

ونظراً لاهمية هذا المخطوط (مارش ٦٢١) التاريخية فقد قمنا باعداده للطبع في مكان آخر واوردنا هنا ملحقاً عربياً يقتصر فقط على هيئة الكواكب العليا ترجماه الى الانكليزية كنموذج لعمل العرضي وكبرهان على كونه هو الواضع لهذه الهيئة وليس قطب الدين الشيرازي .

اما اهمية هذه الهيئة الجديدة التي ابتكرها العرضي فيمكن في كونها اول هيئة تُكتشف الى الآن وفيها يستطيع العرضي ان يرد بشكل ناجح على عيوب هيئة بطلميوس اليوناني . ولتبيان الفرق بين هيئة العرضي وهيئة بطلميوس ارفقنا النص برسوم تبين هيئة الكواكب العليا كما توهمها كل من هذين الفلكيين .

اما الاشكال الوارد في هيئة بطلميوس والذي تمكن العرضي ان يتجنبه فيلخص في هيئة الكواكب العليا في ان بطلميوس جعل مركز فلك التدوير يدور بسرعة مستوية حول مركز جديد غير مركز حامله سمّاه مركز معدل المسير . وهذا مستحيل كما بين ذلك ابن الهيثم في القرن الحادي عشر الميلادي .

اما مخطوط اكسفورد فلا يحوي سوى وصفا لهذه الافلاك وحركاتها . والرسم الوحيد المرفق بالنص اشير اليه على الهامش بعبارة « هذا الشكل خطأ » . لذلك رأينا ان نعيد رسم هيئة هذه الافلاك حسب مقتضيات النص واثبتناها تسهيلاً للقارىء الذي يود تتبع الوصف الهندسي لهيئة العرضي الجديدة .

نخلص الآن الى القول بان الملحق العربي يعطينا لمحة ولو وجيزة عن اعمال العرضي وعن الدور الذي لعبته هذه الاعمال في كتابات الفلكيين الآخرين من امثال قطب الدين الذين لم يذكروا هيئة العرضي فحسب بل رأوا ان يوردوها كاملة في كتبهم ويتبنوها حتى تحسب وكأنها من اعمالهم هم . اضف الى ذلك ان هذه الاعمال الفلكية للعرضي وغيره تشير الى نشاط لم يسبقه مثيل من حيث الاصالة العلمية طوال القرون الوسطى . ولن نتمكن من التعرف على هذا النشاط بشكل دقيق قبل ان يتم لنا استرجاع هذه النصوص ودراستها دراسة علمية وافية .

أبو الوفاء البوزجاني ونظرية ايرُن الاسكندراني

ا. س. كندي و مصطفى موالدي

تحتوي مخطوطة المكتبة الظاهرية بدمشق ذات الرقم – ٤٨٧١ – على عدد من التحقيقات العربية للمقاطع الفلسفية من العصور القديمة ، وقد حقق ونشر العديد منها إن ما تبقى من المخطوطة نفسها يتضمن العديد من الاعمال العلمية والقسم الاكبر منها وحيد وله اهمية تاريخية كبيرة .

وهذه الدراسة تناقش احد نصوص المخطوطة ، وهي دراسة صغيرة تتناول الصفحة رقم / ٨٢ / من المخطوطة .

لقد ذُكر في بداية النص اسم شخصيتين هامتين وكلتاهما معروفة في تاريخ العلوم الدقيقة أولاهما أبو الوفاء البوزجاني (٩٤٠ – ٩٩٨ م) المهندس والفلكي والرياضي (واضع البرهان للمسألة المبحوثة) ُولد في بوزجان وعمل وتوفي ببغداد ، ثانيتهما الفقيه أبو علي الحسن بن حارث الحبوبي . كان الحبوبي معاصراً للبوزجاني ، كما يؤكد ذلك النص المدروس وكذلك ابو ناصر منصور بن عراق حيث يشير الى رسالة ارسلها مع ابي الوفا الى الحبوبي تتضمن بعض التطورات في المثلثات الكروية .

وهذه المسألة استرعت اهتمام العديد من العلماء كأرشميدس وايرُن والبيروني والخــازني وغيرهم وتناولوها بالبحث والدراسة وببراهين عديدة ومختلفة .

وبرهان مخطوطة الظاهرية كان جواب ابي الوفاء البوزجاني عما سأله الفقيه ابو علي الحسن بن حارث الحبوبي عن ايجاد مساحة المثلث بدلالة الاضلاع بدون معرفة الارتفاع ، ويعبر البوزجاني عن نص المسألة كما يلي : [اذا اردنا ذلك ضربنا نصف مجموع ضلعين من اضلاعه (المثلث) اي ضلع كان في مثله ونقصنا من المجتمع مضروب نصف الضلع الثالث في مثله وحفظنا الباقي ثم ضربنا فضل نصف مجموع الضلعين الاولين على احدهما في مثله ونقصنا ذلك من مضروب نصف الضلع الثالث في مثله فيما بقي ضربناه فيما حفظناه اولاً واخذنا جذر المجتمع فما كان فهو مساحة المثلث] ، فبالعودة الى الرسم الموجود في البحث الاصلي صفحة (23) من هــذه المجلة يمكن كتابة العلاقة بالطريقة الحديثة وبالرموز على الشكل التالي :

$$\sqrt{\left[\left(\frac{c+b}{2}\right)^2-\left(\frac{a}{2}\right)^2\right]\left[\left(\frac{c-b}{2}\right)^2-\left(\frac{a}{2}\right)^2\right]}$$

حيث ($c = \overline{AB} , b = AG , a = \overline{GB}$) وهي اطوال اضلاع المثلث

فقد انطلق البوزجاني لبرهان مسألته من الفرضيات التالية :

خذ مثلثاً مـا $\overset{\triangle}{ABG}$ مدد الضلع $\overline{AB}$ الى H بحيث يكون $\overline{AH} = \overline{AG} = b$ ، ونصّف $\overline{BH}$ في Z . و $\overline{BG}$ في E ، واسقط العمود AD على a ، ورسم نصفي دائرة BTZ و BLE قطراهما BZ ، BE على الترتيب .

ورسم الأطوال التالية بحيث تكون على الشكل التالي :

$$\overline{BT} = \overline{BE}$$
$$\overline{EL} = \overline{AZ}$$
$$\overline{BY} = \overline{DE}$$
$$\overline{BK} = \overline{AZ}$$

وللبرهان على مسألته اعتمد على مقدمتين وهما :

المقدمة الاولى : $$\frac{\overline{HB}}{\overline{BG}} = \frac{\overline{DE}}{\overline{AZ}}$$

ولبرهان المقدمة الأولى اعتمد بشكل أساسي على العلاقة التالية .

$$\overline{BA}^2 - \overline{AG}^2 = \overline{BD}^2 - \overline{DG}^2$$

وعلى نظرية فيثاغورث

المقدمة الثانية : $$\overline{TZ}^2 - \overline{YK}^2 = \overline{AD}^2$$

وللبرهان على المقدمة الثانية فقد انطلق البوزجاني من العلاقة التالية :

$$\overline{BZ}^2 + \overline{ZA}^2 = 2\,(\overline{BZ}\cdot\overline{ZA}) + \overline{AB}^2$$

حيث $$\overline{BZ}^2 = \left(\frac{b+c}{2}\right)^2 \text{ ، } \overline{ZA}^2 = \left(\frac{b-c}{2}\right)^2$$

أما البرهان الاساسي لمسألته التي يمكن أن تصاغ كالتالي :

$$(\overline{BZ}^2 - \overline{BE}^2)\,(\overline{BE}^2 - \overline{AZ}^2) = \overline{ABG}^2$$

حيث لدينا وبالاعتماد على نظرية فيثاغورث
$$\left.\begin{array}{l}(\overline{BZ}^2-\overline{BE}^2)=(\overline{BZ}^2-\overline{TB}^2)=\overline{TZ}^2\\(\overline{BE}^2-\overline{AZ}^2)=(\overline{BE}^2-\overline{EL}^2)-\overline{BL}^2\end{array}\right\}$$

وانطلاقاً من المقدمة الاولى :

$$\frac{\overline{HB}}{\overline{BG}}=\frac{\overline{DE}}{\overline{AZ}}\Rightarrow\frac{2\overline{ZB}}{2\overline{BE}}=\frac{\overline{DE}}{\overline{AZ}}\Rightarrow\frac{\overline{ZB}}{\overline{BE}}=\frac{\overline{DE}}{\overline{AZ}}$$

وذلك بالاعتماد على

$$\frac{\overline{ZB}}{\overline{BT}}-\frac{\overline{YB}}{\overline{BK}}\quad(1)\ (\overline{BE}=\overline{BT}\ \text{،}\ \overline{YB}=\overline{DE}\ \text{،}\ \overline{AZ}=\overline{BK})$$

ينتج من العلاقة (1) ان المثلثين $\overset{\Delta}{YKB}$ ، $\overset{\Delta}{ZTB}$ متشابهان

$$\hat{K}=\hat{T}=\text{قائمة}\Rightarrow\overline{YK}\,//\,\overline{TZ}$$

من تشابه المثلثين يمكن كتابة العلاقة التالية :

$$\frac{TZ}{\overline{KY}}=\frac{\overline{TB}}{\overline{BK}}\Rightarrow\frac{\overline{TZ}^2}{\overline{KY}^2}=\frac{\overline{TB}^2}{BK^2}\Rightarrow\frac{\overline{TZ}^2-\overline{KY}^2}{\overline{TZ}^2}\quad\frac{TB^2-\overline{BK}^2}{\overline{TB}^2}\quad(2)$$

واعتماداً على العلاقات التالية :

$$\left\{\begin{array}{ll}\overline{TZ}^2-\overline{KY}^2=\overline{AD}^2 & (\text{المقدمة الثانية})\\ \overline{TB}^2-\overline{BK}^2\quad\overline{BL}^2 & \left(\text{وذلك بتطبيق نظرية فيثاغورث على المثلث } \overset{\Delta}{EBL}\right)\\ \overline{BE}=\overline{BT} & (\text{فرضاً})\end{array}\right.$$

وبعد تعويض العلاقات الثلاث السابقة في العلاقة (2) ينتج لدينا

$$\frac{\overline{AD}^2}{\overline{TZ}^2}=\frac{\overline{BL}^2}{\overline{BE}^2}$$

وبما ان $\overline{AD}$ الارتفاع ولدينا $\overline{BE}=\frac{a}{2}$ اذاً

$$\overline{AD}^2\cdot\overline{BE}^2=\overline{ABC}^2\ \text{مساحة}\ =\overline{TZ}^2\cdot\overline{BL}^2\qquad(3)$$

وبتطبيق نظرية فيثاغورث على المثلث $\overset{\triangle}{ZTB}$ ينتج

$$\overline{TZ}^2 = \overline{BZ}^2 - \overline{BT}^2 \cdot$$

وبما أن $\overline{BT} = \overline{BE}$ اذاً

$$\overline{TZ}^2 = \overline{BZ}^2 - \overline{BE}^2 \qquad (4)$$

وكذلك بتطبيق نظرية فيثاغورث على المثلث $\overset{\triangle}{ELB}$ ينتج

$$\overline{BL}^2 = \overline{BE}^2 - \overline{EL}^2$$

وبما ان $\overline{EL} = \overline{AZ}$ ينتج

$$\overline{BL}^2 = \overline{BE}^2 - \overline{AZ}^2 \qquad (5)$$

وبتطبيق (4) و (5) على (3) ينتج :

$$\overline{\text{مساحة } ABG}^2 \quad . \quad \overline{TZ}^2 \cdot \overline{BL}^2 = (\overline{BZ}^2 - \overline{BE}^2)(\overline{BE}^2 - \overline{AZ}^2)$$

وبذلك نتوصل الى برهان مسألة مساحة المثلث بدلالة الأضلاع للبوزجاني وبشكل مختصر، بينما نجد في القسم الأجنبي من هذه المجلة البرهان الكامل للمسألة مع مناقشة النص العربي وتحقيقه ومقارنة برهان البوزجاني مع حلول أخرى للمسألة نفسها بدءاً من حل أرشميدس .

بقاء علم الفلك العربي في العبرية

ب. غولدستاين

إن المخطوطات العبرية مصدر هام للعلوم العربية ، وهي كثيراً ما تحتوي على نصوص لم تصلنا إلا بواسطتها . ويمكن التمييز بين ثلاثة أنواع للنصوص : ا – نصوص عربية مكتوبة بالحرف العبري ، ب – ترجمات إلى العبرية ح – مقالات عبرية أصلية مبنية على الأصول العربية .

نجد في الفئة الأولى نسخاً عن المجسطي : ملخص لمؤلف مجهول للمجسطي ، وإصلاح المجسطي لجابر بن أفلح ، وكتاب التبصرة في علم الهيئة للخراقي ، والزيج الجديد لابن الشاطر .

ونجد في الفئة الثانية ترجمتين للمجسطي ، وكتاب البطروجي عن مبادىء علم الفلك ، ثم كتاب نور العالم ليوسف بن نحمياس ، والزيج ليوسف بن الوقار ، والزيج للملك ألفونسو ، والزيج أو الغ بك .

ونجد في الفئة الثالثة كتاب الزيج لابراهيم بارحييا المستند على كتاب البتاني ثم كتاب الزيج الشعبي المسمى بالأجنحة الستة لايمانويل بونفيس من تارسكون وقد ترجم هذا العمل فيما بعد إلى اللاتينية واليونانية البيزنطية ؛ ويوجد أيضاً في الفئة الثالثة كتاب الزيج مع لوائح لليوي بن جيرسون ، وهذا العمل يستند إلى نسخ جديدة وهو مأخوذ عن ارصاده الخاصة .

وثمة عملان لهما أهمية خاصة وهما :

١ – نص عربي مجهول مكتوب بالحرف العبري ، ومأخوذ عن النص اللاتيني لكامبانوس من نوفارا (في ايطاليا) ، وهو مثال فريد للنصوص الفلكية المترجمة من اللاتينية إلى العربية .

٢ – واللوائح الفارسية لشلومو بن الياهو من سالونيكي وهي مترجمة من اليونانية إلى العبرية ومستندة في الأساس إلى الزيج السنجري للخازني ، والزيج العلائي للفهاد .

السيمياء الاسلامية وولادة الكيمياء

سيد حسين نصر

ان السيمياء هي في آن واحد علم الكون ، وعلم الروح المقدس ، وعلم المواد ، وعلم متمم لبعض فروع الطب التقليدي . وهي ليست الكيمياء الأولية بالرغم من انها تعالج المواد الطبيعية من وجهة نظر معينة ولا هي أيضاً أصل الطريقة العلمية الحديثة ، بالرغم من أنها اهتمت بأدق معاني التجربة والتجريب ، ذلك أن التجربة الداخلية وحدها هي التي تؤدي الى اليقين وتظل التجربة الخارجية ظلاً باهتاً لها . ويهدف السيميائي التقليدي الى تحويل الطبيعة بحيث يعيدها الى كمالها الأصلي الذي هو من صلب الواقع .

استطاعت السيمياء الاسلامية ان تحتفظ عبر القرون بصفة روحية متكاملة متحدة مع الصوفية ومدارس أخرى فنية ، إضافة الى أنه في الاسلام زُرعت أول بذور علم الكيمياء ، بالرغم من أن النظرة الرمزية للطبيعة السائدة لم تسمح إطلاقاً للنظرة الدنيوية نحو المواد المادية بان تهيمن .

ان ظهور الكيمياء مرتبط بولادة مدرسة فلسفية على هامش الحياة الفكرية الإسلامية وهي متجهة نحو تغيير في وجهة النظر الفكرية التي تتماثل مباشرة مع الفرق الشاسع بين وجهات نظر السيمياء والكيمياء . واكثر من ذلك فإن أحداث هذه المدرسة الفلسفية الهامشية وولادة الكيمياء تعود الى فترة مبكرة من التاريخ الاسلامي وتتعلق باثنين من أشهر الشخصيات في العلوم الاسلامية وهما : جابر بن حيان المسمى باللاتينية جبر (Geber) والذي توفي في القرن الثالث الهجري الموافق للقرن التاسع الميلادي . ومحمد بن زكريا الرازي المسمى باللاتينية رازس (Rhazes) والمتوفي في القرن الرابع الهجري الموافق للعاشر الميلادي .

لم تعرف حوليات السيمياء الاسلامية شخصين ألمع من هذين الرجلين اللذين أظهران عبقرية متعددة الجوانب ، كل منهما كان سيداً شهيراً في السيمياء وتعتقد الأجيال التالية في عالم السيمياء الغربي والاسلامي أن كليهما انتميا الى ذات المدرسة . لكن دراسة حول كتابات كلا الرجلين تظهر بوضوح أنه بالرغم من أن الرازي استخدم لغة السيمياء الجابرية لكنه كان في الواقع لا يعالج السيمياء بل الكيمياء . . . نستطيع أن نقول أن الرازي حول

السيمياء الى كيمياء بالرغم من بقاء السيمياء من بعده زمناً طويلا واستمرار العالم الاسلامي برعايتها .

إتبع الرازي بدقة مصطلحات السيمياء الجابرية ولم يتبنّ من جابر التسميات الفنية فحسب بل تبنى أيضاً عناوين الكتب ، إن عدداً كبيراً من مؤلفات الرازي في هذا المجال تحمل ذات العناوين التي استعملها جابر بينما البعض منها ليس الا تعديلات لأسماء اعمال تعود الى مجموعة جابر الكاملة .

ومن ثم يمكن السؤال لماذا سميت اعمال الرازي بأول كتب الكيمياء في تاريخ العلوم . لدينا عدة أعمال في السيمياء للرازي كالمدخل التعليمي الذي خدم كأساس للقسم عن السيمياء في مفاتيح العلوم . والأكثر أهمية هو كتاب سر الأسرار المعروف في العالم الغربي « Liber Secretorum Bubacaris » وفي كل هذه الأعمال هنالك وصف وتصنيف للمواد المعدنية والعمليات الكيماوية والآلات وغيرها... بحيث يُستطاع ترجمتها بسهولة الى اللغة الكيميائية الحديثة . ليس هناك إهتمام بالوجه الرمزي للسيمياء في مناقشة المعادن وتحولاتها كرموز لتحولات الروح . فالتطابق بين العالم الطبيعي والعالم الروحاني الذي يشكل أساس النظرية العامة للسيمياء قد اختفى معظمها وتركنا مع علم يعالج المواد الطبيعية التي تؤخذ بعين الاعتبار من حيث حقيقتها الخارجية فقط علماً بأن لغة السيمياء وبعض أفكارها ما زالت باقية .

يجب أن نفتش عن سبب خروج الرازي عن النظرة السيميائية في موقفه الفلسفي الخاص كما أننا نعلم في الكثير من المراجع اللاحقة من ضمنها البيروني الذي كان يؤيده علمياً ، نعلم بأن الرازي كتب العديد من الاعمال ضد الدين النبوي وحتى أنه رفض النبوة على هذا الشكل بمفهومها العام وعندما نحلل المواقف الدينية والفلسفية المقتضبة في موقف الرازي نجد السبب واضحاً في تحويله للسيمياء الجابرية الى الكيمياء وفقاً للمفهوم الاسلامي المعد لفئة معينة فقط ان علوم الطبيعة مرتبطة بعلم الوحي . فالوحي له مظهر ظاهر ومظهر باطن وعملية التحقيق الروحي تقتضي البداية من الظاهر والوصول في النهاية الى الباطن . فهذه العملية تسمى بالتأويل ، وإذا طبقنا هذا التأويل على الطبيعة فإنه يعين إختراق ظواهر الطبيعة ليكشف عن كنه الأشياء وهذا يعني التحويل من الحقيقة الى الرمز لرؤية الطبيعة ، ليست الرؤية التي تحجب العالم الروحي بل التي تكشف عنه .

فالسيمياء هي تماماً علمٌ كهذا مبني على اساس الظواهر الطبيعية وبصورة خاصة مملكة المعادن وليس كحقائق بحد ذاتها بل كرموز لمستوى أرفع للحياة .

فجابر بينما كان لو يهتم أيضاً بالحوادث الطبيعية لم يفصل ابداً الحقائق في عالم الطبيعة عن محتواها الرمزي الروحي وميزانه الشهير لم يكن محاولة لقياس مقادير دراسة الطبيعة في مفهومها الحديث بل « لقياس ميول عالم الروح » ان اهماكه بالرموز الأبجدية والرقمية في دراسة الظواهر الطبيعية كتحديد عالم الروح برموز السيمياء بصورة خاصة كلها تشير الى أن جابر كان يطبق عملية التأويل على الطبيعة لكي يفهم معناها الباطن .

فالرازي عند رفضه للتنبؤ وعملية التأويل التي تعتمد عليه برفض أيضاً تطبيق هذه الطريقة على دراسة الطبيعة ، وبهذا حوّل السيمياء الجابرية الى كيمياء ، هذا لا يعني أنه توقف عن استعمال المصطلحات أو الافكار السيميائية ولكن من وجهة نظره لم يكن هناك بعد أي ميزان لقياس ميول عالم الروح أو أي رموز تصبح كجسر بين عالم الظواهر وعالم الأشياء حيث مفاهيمها كما هي في ذات نفسها .

تمت دراسة حقائق الطبيعة كما هي من قبل ولكن كحقائق وليست كرموز وتمت دراسة السيمياء ليس كدراسة السيمياء الحقيقية بل بدراسة كيمياء بدائية فلذلك ارتبط موقف الرازي الديني والفلسفي مباشرة بوجهات نظره العلمية وكان مسؤولاً عن هذا التحول . في الواقع ان حالته تظهر احدى أوضح المثل حيث الأمور الفلسفية والدينية لعبت دوراً في الكثير من التطورات الهامة في العلوم وتاريخ العلوم بصورة عامة، وهي تظهر العلاقة الوثيقة بين وجهة نظر المرء نحو علوم الطبيعة ورؤيته عن الحقيقة كما هو في حد ذاته .

لكن الحضارة الاسلامية رفضت الآراء الفلسفية للرازي وأمثاله وظلت مخلصة لروحها الشعبية الخاصة وعبئها الذي اثقلتها به الأيدي الإلهية أي حمل رسالة القرآن للإنسانية حتى نهاية العالم . سمحت هذه الحقيقة للإسلام بأن يحتفظ الى يومنا هذا بالرغم من كل تغيرات الزمن بمعرفة ومزاولة السيمياء الداخلية التي تجعل من الممكن القيام برعاية الذهب الذي هو هدف الحياة الإنسانية والذي يسمح للإنسان بأن يلعب الدور المرسوم له وأن يعمل كالجسر الواصل بين السماء والأرض وكالعين التي من خلالها يرى الله خلقه وكالمنفذ الذي تعبر الرحمة السماوية من خلاله الى الارض فتخصبها .

مثال حاسم على تأثير مباحث علم النفس في العلوم والحضارة الإسلامية : بعض العلاقات ما بين علم النفس عند إبن سينا وفروع أخرى للفكره والتعاليم الإسلامية

روبرت هول

كانت نظرية علم النفس، هي محور الإهتمام في العالم الاسلامي في العصور الوسطى وكان ابن سينا الشخصية الرئيسية في تاريخ الفكر الاسلامي . ومن ثم نستطيع القول ان علم النفس كان مركز اهتمام ابن سينا و « واسطة العقد » في أعماله حتى أن نظرياته نالت أهمية عظيمة في تاريخ علم النفس . وفي الحقيقة لم يكن لإبن سينا منافس في العصور الوسطى الإسلامية والغربية (وأضف قولي : وحتى في عصر النهضة) لم يكن له منافس سوى ابن رشد (١١٢٦ – ١١٩٨ م) . وأعتقد ان كنت على حق ، أن علم النفس عند ابن سينا أخذ معانٍ ودلالات أبعد في تاريخ الفكر الإسلامي . لأن النظام الفلسفي الذي أبدعه كان نقطة تحولٍ كلي في تاريخ الفلسفة والعلوم والتحقيق النظري - وحتى في التحقيق الديني – في العالم الاسلامي اد دار معظم تفكير ابن سينا حول تحليل العواقب النفسية . لذلك يجب أساساً وقبل كل شيء فهم تعاليم ابن سينا ومذاهبه النفسية فهماً صحيحاً من أجل تحليل تاريخ الفكر الاسلامي أو بالأحرى الفهم السيم لسياق العلوم الاسلامية .

وكان كتاب الشفاء لإبن سينا أطول عرض نظامي متكامل للفلسفة (وأعني بالفلسفة الفلسفة الإسلامية فقط للمفهوم الذي وضعه اليونان) في الفترة الكلاسيكية . ولكن بالرغم فمن من ذلك الممكن أن نقول (حسب وجهة نظر بعض الباحثين المعصرين) أن كتاب الشفاء والأعمال الفلسفية الأخرى لإبن سينا احتوت على تحول جوهري في تقاليد الفلسفة الإسلامية والشاهد على ذلك التهم التي صبّها إبن رشد على إبن سينا لتخليه عن المبادىء الأرسطوطليسية البحتةو الفلسفةالروحيةالبحتة التي إستطاع نصير الدينالطوسي (١٢٠١– ١٢٧٤) أن يحدثها من خلال تفسير بن سينا في شرح الإشارات وفي كتب أخرى .

إن السياق الفسفي الذي ضم في القرنين التاسع والعاشر المنطق والرياضيات والفلسفة الطبيعية والعلوم الطبيعية على المبدأ الرياضي وعلم ما وراء الطبيعة وعلم الأخلاق والسياسة إن هذا السياق إحتفظ بشيء من نظرية أرسطو الأصلية للبحث والتطور التصاعدي للمعرفة ولكن فيما بعد أصبح ذلك دراسة تمهيدية بحتة ولو أنها أساسية لنوع من المعرفة

الإستشراقية المباشرة وعلى وجهه الإفتراض أكثر قيمة وأصبحت في آخر الأمر تفسر في المدارس الإيرانية الحديثة بالمعرفة الروحية بالرغم من كوني لا أستطيع أن أكشف عن مذهب باطني حقيقي لا في أعمال إبن سينا – وبالتأكيد – ولا في الفصل المستشهد منه مراراً عن مقامات العارفين في كتاب الإشارات مع ذلك كانت المقومات الإشراقية بارزة بوضوح وكانت الأرض ممهدة تماماً للتطور الروحي بفضل فلسفة إبن سينا .

وأنا متأكد أن القوة المحركة وراء هذا التحول للفلسفة مستمدة من البحث الفلسفي عـن الروح أو بالأحرى عن المعاني المتضحة التي تعطيها مبادىء علم النفس في كل مجالات التحقيق الفلسفي تقريباً . إن ذات النتائج النفسية الأساسية ذاتها أدركها في النهاية المتكلمون (علماء الدين الأسلامي الذين يبحثون وفق البراهين العقلية) كما كانوا يسمون بعض الصوفيين ذوي الميول الفكرية وبالفعل كل المسلمين المثقفين في ذلك العصر إن المهمة الأساسية في تطور الفكر الإسلامي الكلاسيكي كانت توسيع النمط النظري للحضارة الدينية المبنية على القرآن . وبعد ذلك فليس من المستغرب وجود مناقشة عامة نادراً ما تقع بين الفئات المتضادة للمفكرين المسلمين كالقضاة المصحيين ، وعلماء الدين (المتكلمون) والفلاسفة والصوفيين والإسماعيليين وإن لهذه المناقشة تأثير توجيهي عظيم على الحضارة هذه المناقشة التي كانت كثيراً ما تنصب على أمور علم النفس . إن السؤال عن الروح ومشاكل المعرفة الصحيحة والإيمان الحق ووثيقة الإرتباط بهعدا الإهتمام الرئيسي وربما الأساسي الأكبر للمفكرين المسلمين .

(ثم يتابع المؤلف في توضيح طريقة إبن سينا ونتائجه بالفحص المفصل بدقة لمعالجة لكلتا المشكلتين المنفصلتين ، مشكلة علم الأجنة ومشكلة الأساس التجريبي للمعرفة . إن حل المشكلة الأولى هو تعديل للحل الذي قدمه أرسطو وأما حله للثانية فهو حل يتعارض جذرياً مع حل أرسطو . وإن عرض هذه الأمور يأخذ معظم البحث وهو مدروس بدقة وفنية إلى درجة عالية غير ملخص هنا . نرجو من قارئنا المهتم أن يعود إلى الأصل الإنكليزي. أما الإستنتاجات فهي فيما يلي. (المحررون) .

لا نستطيع أن ننكر ذكاء أو على الأقل دقة التأليف الفلسفي الذي أنجزه ابن سينا . لقد قدم حلولاً للمسائل النفسية الأساسية التي كانت تواجهه وحتى إذ أنه ترك مجموعة من الأسئلة الثانوية في علم الوجود دون جواب . بإستخدامه لطريقة عرض غير مباشرة ممتازة قدم إبن سينا تفسيراً لا يتوافق مع تعاليم أرسطو عن إكتساب المعرفة وهذ التفسير ترك إبن سينا في تمام الموقف الإشراقي المعتدل الذي أراده . لقد كان تحليل التجربة في كتاب البرهان خطوة حاسمة في تثبيت مبادئه النظرية للمعرفة . من الممكن ان تكون التجربة ذات فائدة ولكن كان لها دور محدود جداً ولم يكن هناك مثال حيث لا يستطيع تجنبها في النهاية . التجربة تعود إلى القدرة الاستنتاجية والمعرفة إلى الفكر ، والأساس الفعال للتفكير يكمن في مكان سماوي. إن إقامة إبن سينا لنظرية التفكير الخارجي للمعرفة كانت الأكثر حسماً . ولقد ناقشت في تحديد العلاقة التالية للعلوم اليونانية وأنصارها مع أتباع الطرق الأخرى للمعرفة في الأسلام .

لقد نسبت موقف إبن سينا من المعرفة التجريبية إلى إعتماد المسلمين الجوهري في الخلاص الشخصي . وإلى هذا أيضاً نسبت تفسير نفخ الروح في الجنين الأنساني المعروض في كتاب الحيوان . أخيراً وبما أن الفردوس كان سيقدم التفكير الخالد كأعظم مكافأة كان من هذا المضمار ولادة مشاكل علم الوجود الرئيسية . لقد قام إبن سينا بخطوات مختلفة ليوفق ما بين التفكير الواقعي مع التمييز الروحي ولكنه لم ينجح نجاحاً حقيقياً .

إن المناقشات في هذا البحث يجب أن تكون قد وضحت العلاقة الكبيرة بين فلسفة إبن سينا وعلم النفس عنده والإعتماد الحاسم لنظامه على تطوير نظرية الروح العامة المستقيمة علاوةً على ذلك إن العالم الفكري الإسلامي في القرن التاسع المتقدم والعاشر وبداية القرن الحادي عشر يصح القول أن نسبة عالية من النتائج الرئيسية كمنت في نظريات علم النفس أو المأخوذة عن مبادئها مباشرةً وإن هذا إصرار يجب أن تزود له قائمتين الأولى حالة ظاهرية كافية . لقد قدمت في وبدون إثبات تحليل لتطور الحضارة الفكرية الإسلامية حيث العملية الأساسية هي حوار بين فئات متعددة للمفكرين المسلمين وإحداها ضمت الفلاسفة وآخرين يحبذون العلوم اليونانية . ويعتقد هنا أنها كانت مناقشة دينية في الأساس وكان السؤال الذي يشكل الأساس هو نوع العلم الذي كان يقبل بأنه صحيح وكان بذلك يؤمن الفهم الصحيح للدين . إن الفحص الكامل للتجربة والأمور المتعلقة بها فيما سبق كان

مقصوداً من ناحية لتوضيح هذه الصورة للمناقشة العامة ولعرض مثال جديد بالذكر لما استنتجت أنه كان متعلقاً به . وإذا كان هذا التفسير صور الوضع التاريخي بشكل دقيق عندئذ يصح القول التالي : أن من خلال هذا الحوار مارست نظرية علم النفس قوة رئيسية على التشكيل النهائي للحضارة الفكرية الإسلامية .

لا يشك أحد في أن ابن سينا كان شخصية رئيسية في تاريخ الفكر الاسلامي . والمهمة الحقيقية هي معرفة بأي وسيلة إستطاعت فلسفة ابن سينا تغيير مسلك العلوم اليونانية في العالم الإسلامي وبذلك غيرت تطور حضارة الإسلام ككل . وهنا جواب غير نهائي ممكن بعد لتحويل الفلسفة بحد ذاتها الى عملية مباشرة نسبياً وأمر يعتمد في الاساس على الإجابات المبدئية وعندما أصبح تركيز الفلسفة على علوم ما وراء الطبيعة والعلوم الرياضية أقل بكثير . ولكن فكرة ابن سينا كأول مثال لهذه الفلسفة كانت قد إستطاعت أن تلعب دوراً رئيسياً إلى جانب الفروع القديمة في الحوار الديني العام التي إفترضتها . وإذا كان الوصف صحيحاً حتى الآن نستطيع أن نؤكد أن علم النفس النظري لابن سينا مارس تأثيراً حاسماً على تاريخ العلوم اليونانية في الإسلام وعلى تطور التاريخ الحضاري في الإسلام عامة . وهذا الإستنتاج هو ما كان قلقاً على إثباته ولكن حتى في بحث يميل إلى الطول من الممكن إعطاء الإثبات الكافي لجزء واحد فقط للمناقشة الضرورية ورغماً من ذلك آمل أن أكون قد عالجت في الأساس تلك النقاط فقط ذات الأهمية الكبرى لنتائجه العامة .

ولأضف ملاحظة نهائية : إذا كان هذا التخمين التاريخي تخميناً دقيقاً كان للفلسفة وكل العلوم اليونانية الصداوة في مركز الحضارة الإسلامية بحد ذاتها وليس على أطرافها كما كان يعتقد غالباً. بالفعل إن الفلسفة وأخواتها الفروع الأخرى ستحتاج أن نعتبرها تطوراً في طرق وعمليات مألوفة في معظم المجالات والمساعي الفكرية في العصور الوسطى الإسلامية والتي هي من ضمن الصفات الأكثر أهمية وتمييزاً في الحضارة الإسلامية .

مقالـه قصيرة واعلانات

الاشارة الى مخطوطة أخرى لكتاب المنصوري للرازي

غادة كرمي

أحد أشهر كتب الرازي (أبي بكر محمد بن زكريا الرازي) هو كتابه الشامل عن الطب الـذي أهداه الى الامير الساماني أبـي صالح المنصور أبي اسحاق والذي عرف فيما بعد بكتاب المنصوري. وكان عملا مشهورا في العالم اللاتيني الغربي خلال العصور الوسطى وترجم الى العبرية واليونانية واللاتينية وقد ترجمه الى اللاتينية جيرارد أوف كريمونا في عام ١١٧٥ ، وقد طبع باللاتينية في عام ١٤٨١ وأعيدت طباعته مرات عديدة فيما بعد. وهنالك الكثير من المخطوطات اللاتينية الاخرى الموجودة عن هذا الكتاب، وهي دليل آخر على شعبية هذا الكتاب في الغرب . ان كتاب المنصوري ينقسم الى أبحاث أو مقالات . والمقالة التاسعة او الكتاب لمنصوري التاسع الذي يبحث في الامراض من الرأس الى الكعب كانت وبصورة خاصة شائعة الاستعمال في القرن الخامس عشر وعلق عليها في القرنين الخامس عشر والسادس عشر. أشهر هذه التعليقات كانت الصياغة الجديدة لأندرياس فيزاليوس التي نشرت في عام ١٥٣٧ .

ولقد كان كتاب المنصوري شائعاً وهاما في الشرق العربي . لقد قال أبو العباس المجوسي مؤلف الموسوعة العلمية : كامل الصناعة في القرن العاشر قال في مقدمته أن الرازي قد تجاوز كل الاخرين بتفوق في كتابه هذا . فاليوم لا توجد أقل من ٤٧ مخطوطة عن هذا العمل الموجود وهي مشتتة موزعة على المكتبات الشرقية والغربية المختلفة . فالعدد الكبير والامتداد الزمني الواسع للمخطوطات الباقية هو دليل آخر على شعبية هذا الكتاب ومع ذلك لا يوجد تحقيق باللغة العربية لهذا العمل في العصر الحديث ما عدا تحقيق رايسكي بالعربية واللاتينية في عام ١٧٧٦. ان المقالة الاولى حققت وترجمت الى الفرنسية من دو كونيج في مطلع هذا القرن .

ان كتاب المنصوري متوسط الحجم (فطول المخطوطة يتراوح عند الـ ٢٢٠ ورقة) وهو يعالج كل المواضيع الكبرى ذات الاهمية الطبية كما تبين مواضيع مقالاته العشر :

شكل الاعضاء ومظهرها

معرفة مزاجات الجسم والاخلاط الراجحة فيها

وظائف الطعام والدواء
الاحتفاظ بالصحة
المستحضرات التجميلية والامراض الخارجية
ادارة المسافر
تجبير العظام والجروح والقروح (التقرحات)
السموم ولسع الحشرات
الأمراض من الرأس إلى الكعب
الحميّات ، المغليات ، النوبات ، البول والنبض

ومن ضمن المؤلفات الطبية كانت مخطوطة كتاب المنصوري فالنسخة الوحيدة لهذا الكتاب والتي عرفنا بوجودها في حلب ، كانت النسخة المذكورة في قائمة الاب بول سباث والمؤلفة من ثلاثة علدات للمخطوطات الموجودة في المجموعات الخاصة في حلب . وهنا يشير الى ان مخطوطه لكتاب المنصوري موجود في مجموعة قنصل هولندا السيد رودولف بوخي . ان التدوين في كتاب سباث مختصر على نحو مميز ولا يعطي اي وصف للمخطوطة. ان التحقيق الدقيق أثبت ان مخطوطة القنصل الهولاندي هي ذاتها المهداة الى معهد التراث لقد انتقلت من ملكيته الى ملكية آخرين ومن ثم الى آخر مشتّري وهو الذي اهداها الى معهد التراث . ومع المخطوطات جاءت أيضاً قائمة مكتوبة بخط اليد فيها عناوين الكتب المخطوطة وأسماء مؤلفها ووصف قصير كل هذا موجود في تمارين صغير ويقال ان بول سباث كتبه في الثلاثينيات من هذه القرن كتحضير لقائمة أشمل لم يقم بها ابداً . وكتاب المنصوري يؤرخ نسخة بالقرن الثالث عشر ميلادي ، ولا يعطي اي شرح آخر . ان وجود هذه المخطوطة بالرغم من انها ذكرت في قائمته لايشار الى وجوده في اي من كتب بروكلن او سيزكين او اولمان .

المخطوطة
(الرقم : انطاكي ١)

الغلاف الجلدي بهت لونه وتعرض للتلف لقد انفصل التجليد ومعظم الاوراق منفصلة ولكن ما عدا ذلك فالمخطوطة محفوظة بصورة جيدة . صفحات عليها بقع تقريبا بدون ملاحظات على الهامش . الصفحة الاولى تحتوي خط المالك وأربع تدوينات بأيد مختلفة احداها وهي تبدو أحدث في النص تقرأ كما يلي :

« كتاب المنصوري في حفظ الصحة ومعالجة الامراض لمن يحضره الطبيب تأليف الشيخ الحكيم أبي بكر محمد بن زكريا الرازي . »

١٨٢ صفحة كاملة . مرقمة باحبر تنتهي عند الصفحة ٣٦٤

١٨,٥ × ١١,٥ سم ٢٢ سطر

الخط نسخي واضح مشكل جزئيا . العناوين بالحبر الاحمر لا يوجد اسم الناسخ . بدون تاريخ ربما القرن السابع / الثالث عشر (كما عند سباث)

وهي تبدأ :

« بسم الله الرحمن الرحيم »

بسم الله الرحمن الرحيم رب يسر وأعن برحمتك مجدا كتاب الفه « محمد بن زكريا » للمنصور بن اسحاق اسمعيل بن احمد فقال اني جامع للامير أطال الله بقاه جملا وجوامعا ونكتاً وعيونا من صناعة الطب ومتحذ في ذلك الاختصار والايجاز وذاكر ما لا يحدث ...

وتنتهي :

فليوخذ لهم رطل من وزن درهم مصطكي ودرهم سنبل قصير في خرقة وتلقى عند الطبخ فيه ان شاء الله تعالى واذا اتينا على جميع المقالات والفصول المذكورة في صدر هذا الكتاب فقد كمل كتابنا هذا والله المعين والموفق للصواب وهو حسبنا ونعم الوكيل ولا حول ولا قوة الا بالله العلي العظيم تم الكتاب والحمد لله حق حمده وصلى الله على سيدنا محمد وآله وصحبه وسلم تسليما .

بالطبع انه من المفيد دائما ان نكشف عن مكان مخطوطة علمية عربية ولكنه من الاهمية الخاصة في هذه الحالة ترجع الى سببين :

اولا : لأنه لا توجد طبعة حديثة لكتاب المنصوري . ثانيا أن هذا الكتاب ذو أهمية عظيمة لتاريخ الطب العربي وتاريخ العلم في العصور الوسطى . بالاضافة الى ذلك ان هذه المخطوطة ذات قيمة خصة لانها كاملة وبحالة جيدة ويبدو انها متقدمة . ان الكثير من المخطوطات (المتبقية) لكتاب المنصوري ليست كاملة وفي بعض الاحيان تفتقد الى نصف او ثلث النص الاصلي .

من حسن الحظ تخلت الملكية الخاصة عن هذه المخطوطة وأصبحت متوفرة لاستعمال الباحثين

برعاية السيد الرئيس حافظ الاسد

انعقدت في جامعة حلب

النـدوة العالمية الثانية لتاريـخ العلوم

عنـد العرب

تحت رعاية السيد الرئيس ــ حافظ الأسد ــ رئيس الجمهورية احتفل بافتتاح الندوة العالمية الثانية لتاريخ العلوم عند العرب ، وقد ناقشت الندوة وعلى مدى خمسة ايام عشرات الأبحاث الاصيله التي قدمها حوالي ١٢٧ ــ عالماً وباحثاً شاركوا في الندوة كما نظمت الندوة حلقة علمية حول تاريخ الجبر العربي واخرى حول انتقال العلم العربي إلى الغرب اللاتيني بالإضافة الى عدد من المعارض هي : معرض الأدوات الفلكية والصناعات الحربية ، معرض مسح المنشآت المائية في القطر ، معرض النباتات والمواد الطبية ، معرض منشورات معهد التراث العلمي العربي ومطبوعات جامعة حلب ، معرض لبعض القطع الأثرية التي تشكل نواة متحف العلم والتكنولوجيا الذي يعمل معهد التراث على احداثه . كما تم خلال انعقاد الندوة عرض فيلم سينمائي عن مدينة (ايبلا) وفيلم آخر عن ابن النفيس ، وبعض الأفلام الأخرى عن العلم في العالم الاسلامي ؛ ونظمت الجامعة لضيوف الندوة برنامجاً تضمن اطلاعهم على الثروة الأثرية العريقة للقطر .

وسيصدر معهد التراث العلمي العربي عدداً خاصاً من رسالته يخصص للندوة العالمية الثانية لتاريخ العلوم عند العرب .

فوز الدكتور فؤاد سزكين
بجائزة الملك فيصل للدراسات الاسلامية

فاز الاستاذ الدكتور فؤاد سزكين في جامعة فرانكفورت في المانيا الاتحادية ومرشح معهد التراث العلمي العربي بجائزة الملك فيصل للدراسات الاسلامية لعام ١٩٧٩ عن مؤلفه « تاريخ التراث العربي » .

وتجدر الاشارة لبيان قيمة هذا المؤلف المنشور باللغة الالمانية ان نذكر ما قاله احد المستشرقين في مؤتمر عقد بمدينة وورزبرغ في المانيا الاتحادية عام ١٩٦٨ «اذا كان كتاب بروكلمان قد حول الأنظار اليه سنوات طويلة فان كتاب « تاريخ التراث العربي » سوف يكون كتاب القرن العشرين في الثقافة العربية وتصنيف التراث العربي الضخم » .

ويعلق الدكتور فهمي ابو الفضل على المؤلف فيقول « ان هذا السفر ليس سفراً للعلوم فقط ، ولكنه سفر لعمل متواصل ومجهود ضخم واذا قلنا ان فرداً واحداً قد قام بعمله ، فربما تطرق الشك الى نفوسنا ، لأنه يجب ان يكون عملاً جماعياً . ولكن الواقع غير ذلك فهو عمل فردي ، بذل صاحبه اكثر من عشرين عاماً في جمعه وتنسيقه وترتيبه حتى ظهر في الصورة التي بين ايدينا . »

وُلد الدكتور فؤاد سزكين في مدينة استنبول عام ١٩٢٤ وحصل على دكتوراه في العلوم الاسلامية والدراسات الايرانية . ومارس التدريس في جامعة استنبول سنوات عديدة عكف خلالها على الاطلاع على كنوز التراث الاسلامي . انتقل عام ١٩٦٠ الى المانيا الغربية حيث تولى التدريس بمعهد اللغات السامية في جامعة ماربورغ لمدة سنتين ثم انتسب الى معهد تاريخ العلوم الطبيعية في جامعة فرانكفورت كاستاذ زائر . ثم اصبح استاذاً بكل الحقوق المعترف بها للأساتذة الألمان رغم احتفاظه بجنسيته التركية الى اليوم .

ومن اهم مؤلفات الدكتور سزكين بالاضافة الى موسوعته « تاريخ التراث العربي » ، كتاب « تاريخ البلاغة » (باللغة التركية) عام ١٩٤٨ و « مجاز القرآن » لأبي عبيدة معمر بن المثنى (مجلدان) نشر بالقاهرة عام ١٩٦٢ و « دراسات حول مصادر الجامع الصحيح البخاري » (باللغة التركية) طبع باستنبول عام ١٩٥٦ .

هذا وقد منحت الجائزة للدكتور سزكين في احتفال رسمي كبير اقيم في الرياض في السابع والعشرين من شباط ١٩٧٩ . وقد دعي لحضور هذا الاحتفال الاستاذ الدكتور أحمد يوسف الحسن رئيس جامعة حلب ومدير معهد التراث العلمي العربي .

وتتكون الجائزة من شهادة تحمل اسم الفائز وملخصاً للعمل الذي أهله لها ومن ميدالية ذهبية ثمينة ومبلغ نقدي قدره ٢٠٠ ألف ريال سعودي .

حفـلة تكريم

الأستاذ الدكتور محمد يحيى الهاشمي

درج معهد التراث العلمي العربي بجامعة حلب على تكريم العلماء والباحثين وخصوصاً العرب منهم ، فاقام المعهد حفلة لتأبين الاستاذ الدكتور احمد شوكت الشطي اثناء الندوة العالمية الثانية لتاريخ العلوم عند العرب واصدر عدداً خاصاً من نشراته جمع فيه الكلمات التي ألقيت خلال تلك الحفلة وكذلك اقام معهد التراث العلمي العربي والجمعية السورية لتاريخ العلوم في جامعة حلب مساء الخميس ٧ - ٦ - ١٩٧٩ حفلاً تكريمياً كبيراً للاستاذ الدكتور محمد يحيى الهاشمي احد علماء حلب المعروفين تقديراً لجهوده واعماله وبحوثه العلمية ومساهمته الفعالة في المؤتمرات الدولية العديدة وتأليفه الكثير من الكتب والقاء المحاضرات وغير ذلك من البحوث .

وقد حضر الاحتفال عدد كبير من رجال الفكر والثقافة واعضاء الجمعية السورية لتاريخ العلوم وافراد اسرة الدكتور الهاشمي .

ولد في حلب سنة ١٩٠٤ من عائلة حلبية عريقة بالفضل والعلم .

- درس في ألمانيا ونال منها شهادة في الكيمياء والفلسفة ، وحصل على الدكتوراه في الكيمياء بتقديمه دراسة عن « كتاب الاحجار للبيروني » .
- درس في مدارس حلب الثانوية ومن ثم في جامعتها الى أن أحيل الى التقاعد .
- كانت حياته نضالاً مستمراً في سبيل احياء علوم العرب والاسلام وتعريف الغرب بها .
- كتب الكثير من المقالات ، واشترك في كل مؤتمرات تاريخ العلوم ، ونشر العديد من الكتب . منها كتاب الامام جعفر الصادق ، ملهم الكيمياء في طبعته الاولى والثانية .
- ومن أبرز أعماله تأسيسه في حلب سنة ١٩٥٧ « جمعية الابحاث العلمية » التي كان لها أثر كبير في سورية ولا سيما في البلاد الغربية بفضل منشوراتها وابحاثها .

مراجعات الكتب

مراجعة « كتاب الحيل » لبني موسى بن شاكر

الترجمة الانكليزية مع التعليق والشرح
دونالد هيــــل
شركة رايدل للنشر – هولندة ١٩٧٩*

عاش بنو موسى في القرن الثالث الهجري / التاسع الميلادي عندما كانت الحضارة العربية الاسلامية في أوجها. وقد لعب الأخوة الثلاثة محمد واحمد احسن ابناء موسى بن شاكر في عهد المأمون ومن تلاه من الخلفاء دوراً بارزاً في تطوير العلوم وبصورة خاصة العلوم الرياضية والفلكية والميكانيكية من خلال مؤلفاتهم ومن خلال تأثيرهم الفعال على حركة الترجمة من اليونانية الى العربية . ورغم كثرة ما الفه بنو مــوسى الا ان اهم ما كانوا يتميزون به هو كتاب الحيل . ولم يرد ذكر لبني موسى الا وكان كتاب الحيل ابرز ما يوصفون به .

وفي مفاتيح العلوم[١] نجد ان الخوارزمي يعتبر علم الحيل واحداً من العلوم الثمانية الرئيسية ثم انه يقسم هذا العلم الى فرعين : الاول جر الأثقال بالقوة اليسيرة والثاني حيل حركات الماء وصنعة الاواني العجيبة وما يتصل بها من صنعة الآلات المتحركة بذاتها . وفي التقسيمات المتأخرة لتفرعات العلوم اصبح علم الحيل احد فروع علم الهندسة ليس بمعناه الرياضي (geometry) بل بمعناه التكنولوجي (engineering)[٢] .

وعلى اي حال وبغض النظر عن تقسيمات العلوم وتباينها من عصر الى عصر فان علم الحيل يدخل في نطاق الهندسة الميكانيكية اذ انه يبحث في الالات والادوات والتجهيزات الميكانيكية والهيدروليكية .

* بالنسبة للاسماء الأجنبية : ارجع الى النسخة الانكليزية

١ – محمد بن أحمد بن يوسف الخوارزمي . مفاتيح العلوم (القاهرة، ادارة الطباعة المنيرية، ١٣٤٢هـ)، ص ١٩١

٢ – أحمد القلقشندي . صبح الأعشى (القاهرة ، المطبعة الأميرية ١٩١٣)، ج. ١ ص ٤٧٦ .

والى عهد قريب اشتهر كتابان فقط في علم الحيل عند العرب احدهما كتاب الحيل لبني موسى والثاني كتاب الجامع بين العلم والعمل النافع في صناعة الحيل لبديع الزمان بن الرزاز الجزري[٣] . ثم اضيف اليهما كتاب ثالث هو كتاب الطرق السنية في الالات الروحانية لتقي الدين بن معروف الراصد الدمشقي[٤]. وبذلك اصبحت هذه الكتب الثلاثة التي تعود الى عهود متباعدة : كتاب بني موسى في القرن الثالث الهجري / التاسع الميلادي . وكتاب الجزري في القرن السادس الهجري / الثاني عشر الميلادي ، وكتاب تقي الدين في القرن العاشر الهجري / السادس عشر الميلادي تشكل حلقات اساسية في سلسلة من تقاليد الهندسة الميكانيكية في الحضارة العربية الاسلامية . وربما اكتملت حلقات هذه السلسلة باكتشاف ونشر كتب اخرى في هذا المجال[٥] .

تبدأ اذن التقاليد العربية الاسلامية في علم الحيل بكتاب بني موسى الذي اكتسب شهرة كبيرة في المراجع العربية . ومن حسن الحظ ان كتاب الحيل هو من الكتب القليلة التي وصلت الينا من اعمال بني موسى ولكن رغم شهرة الكتاب فان المخطوطات المتبقية منه قليلة . وهناك الآن ثلاثة مخطوطات رئيسية منه فقط هي مخطوطة طوبقابي سراي . احمد الثالث ٣٤٧٤ ومخطوطة الفاتيكان رقم ٣١٧ ومخطوطة ثالثة موزعة بين مكتبة غوتا في المانيا الديموقراطية وتحمل الرقم ١٣٤٩ ــ أ (1349 a) وبين مكتبة برلين في المانيا الغربية وتحمل الرقم ٥٥٦٢ . والمخطوطة الاولى (طوبقابي) لم تكتشف الا مؤخراً[٦] .

بدأ اهتمام مؤرخي العلوم بكتاب الحيل لبني موسى منذ نهاية القرن الماضي ولكن الدراسات الجادة حوله بدأت في مطلع هذا القرن عندما نشر كل من ڤيدمان وهاوسر مقالات حول اواني الشراب وشرحا الاشكال ٨٥ ــ ٨٧ من كتاب الحيل[٧] . ثم نشر

٣ ــ صدر النص العربي مؤخراً : الجامع بين العلم والعمل النافع في صناعة الحيل لابن الرزاز الجزري ، تحقيق الدكتور الحسن وزملاؤه (معهد التراث العلمي العربي ١٩٧٩) وسبقته الترجمة الانكليزية لـ دونالد هيل ، ١٩٧٣.

٤ ــ صدر النص العربي لأحمد يوسف الحسن « الطرق السنية في الآلات الروحانية » ، معهد التراث العلمي العربي ــ حلب ، ١٩٧٦ .

٥ ــ نشر لـ دونالد هيل في مجلة تاريخ العلوم العربية ١ (١٩٧٧) ، ٣٣ ــ ٤٦ . مقالة عن كتاب الأنسي في الآلات يعود الى القرن الخامس الهجري / الحادي عشر الميلادي ، وثبت فيها بعد انه المرادي .

٦ ــ انظر مجلة هيل الجزري ترجمة دافيد ا. كينغ « الحيل في العصور الوسطى » ، تاريخ العلوم ١٣، (١٩٥٧)، ٢٨١ ــ ٢٨٩ .

٧ ــ ايلهارد ڤيدمان و ف. هاوسر « الجزري و بنو موسى » في مجلة الاسلام ، ٨ (١٩١٨) ص ٥٥ــ ٩٣ ، ٢٦٨ ــ ٢٩١ .

هاوسر كتاباً موسعاً ادرج فيه بقية اشكال كتاب الحيل[8] . وبذلك اصبح كتاب الحيل معروفاً باللغة الالمانية وقد استند فيدمن وهاوسر الى مخطوطة الفاتيكان بصورة رئيسية والى مخطوطة غوتا – برلين بصورة ثانوية . ونظراً للنواقص الكثيرة والاخطاء الواردة في هاتين المخطوطتين فقد بذل هاوسر جهداً كبيراً في محاولة تفسير الاشكال ولم يتقيد بسبب ذلك بايراد ترجمة حرفية للكتاب بل اعاد الصياغة بالالمانية بالاسلوب الذي يجعل النص مفهوماً من الناحية الفنية .

وكان العمل الاخير والهام الذي تناول كتاب الحيل هو الترجمة الانكليزية الكاملة التي صدرت هذا العام والتي قام بها دونالد هيل . وهيل بعمله هذا يكمل ما كان قد بدأه عندما أصدر الترجمة الانكليزية لكتاب الجزري في عـــام ١٩٧٥ بالاضافة الى اعمال اخرى قام بها ونشرها (أو هي في سبيل النشر) عن التكنولوجيا الميكانيكية العربية الاسلامية.

ويتميز كتاب الحيل لبني موسى الذي اصدره هيل بالانكليزية بانه اول كتاب يصدر مشتملاً على كامل كتاب الحيل باية لغة كانت بما في ذلك اللغة العربية . وقد كان لاكتشاف مخطوطة طوبقابي في استانبول اهمية كبيرة زادت من قيمة كتاب هيل وجعلته متميزاً عن كتاب هاوسر الصادر باللغة الالمانية .

قسم هيل كتاب الحيل الى قسمين ، القسم الاول هو المقدمة واهم ما اشتملت عليه هو : ١ – حياة بني موسى واعمالهم ٢ – مخطوطات كتاب الحيل مع تحليل مفصل قارن فيه بين المخطوطات الثلاثة الرئيسية ، ٣ – الابحاث السابقة التي تناولت كتاب الحيل ، ٤ – تحليل تاريخي لكتاب الحيل والاعمال المماثلة ، ٥ – شرح للمبادىء والوسائل الاساسية التي استخدمت في تصميم تجهيزات بني موسى في كتاب الحيل . والقسم الثاني من الكتاب يحتوي على الترجمة الكاملة للاشكال (Devices or models) المائة لكتاب الحيل مع الملاحظات والتعليقات في نهاية كل شكل .

واورد هيل بعد ذلك ملحقاً يحتوي على ثلاثة اشكال لم تثبت نسبتها الى كتاب الحيل وقد ورد احدها في مخطوطة الفاتيكان والثاني في مخطوطة طوبقابي والثالث في مخطوطة ليدن (Or 168) . واورد هيل بعـــد ذلك قائمة بالمراجع ثم اورد معجماً Glossary بالمفردات العربية ومعانيها باللغة الانكليزية .

٨ – ف. هاوسر : عن كتاب الحيل ، (ارلونغن ، ١٩٢٢) .

وقد استخدم هيل مخطوطة طوبقابي بشكل رئيسي وحيثما كان النص موجوداً في هذه المخطوطة فقد كانت هي المعتمدة وقارنها مع مخطوطة الفاتيكان ولم يجد ضرورة للمقارنة مع مخطوطة غوتا – برلين . وفي حالات اخرى كانت الفاتيكان هي المخطوطة المعتمدة وذلك بالنسبة للاشكال المفقودة من مخطوطة طوبقابي . وفي الاشكال العشرة الاخيرة أصبحت مخطوطة برلين هي المخطوطة الوحيدة نظراً لفقدان هذه الاشكال من كل من مخطوطتي طوبقابي والفاتيكان .

وقد اورد هيل في نهاية كل شكل Model الصورة الفوتوغرافية للرسم الخاص بذلك الشكل كما ورد في المخطوطة ، ثم اعاد رسم ذلك الرسم من جديد مهملاً التفاصيل غير الضرورية كايدي الاباريق والزخارف وغيرها ودون على هذه الرسوم (التي اعاد رسمها) الحروف اللاتينية المرادفة للحروف العربية . وفي بعض الحالات اورد ايضاً رسومات توضيحية حديثة واقتبس بعضاً من هذه الرسوم التوضيحية من كتاب هاوسر مشيراً الى ذلك في جميع الحالات .

واورد هيل في نهاية كل شكل الملاحظات اللازمة لشرح الافكار الغامضه او لتوضيح المبادىء التي يستند اليها عمل ذلك الشكل . ولكنه احتصر الكثير من الشرح عندما اورد في مقدمة الكتاب فصلاً خاصاً شرح فيه المبادىء والوسائل التي استحلمها بنو موسى في تصميم اشكالهم والتي تكررت في تلك التصاميم .

ان هذا العمل الذي قام به هيل جدير بالاحترام والتقدير . ويدرك ذلك كل من حاول تحقيق كتاب من هذا النوع . فهو يحتاج الى خبرة ودراية بالفن ذاته كما انه يحتاج الى معرفة جيدة باللغة العربية . ولقد تخصص هيل بابحاث انفرد بها واكسبته شهرة استحقها بجدارة عندما ركز اعماله على ترجمة المخطوطات الخاصة بالتكنولوجيا الميكانيكية العربية الاسلامية . واصدر حتى الآن أهم كتب الحيل العربية باللغة الانكليزية قبل ان تصدر هذه الكتب باللغة العربية ذاتها. وفي عمل علمي صعب من هذا النوع لا يخلو الأمر من ورود بعض الاخطاء ، ولكن هذه الاخطاء تصبح ثانوية وغير ذات قيمة نسبية امام الاهمية الكبيرة لهذا الانجاز . لقد اعطى هيل بني موسى حقهم كاملاً بعمله هذا ولم يعد كتاب الحيل اسطورة تتداول بشأنها ما اورده ابن النديم والقفطي وحاجي خليفة بل اصبح الآن كتاباً علمياً هندسياً نفهمه ونتمتع بقراءته . والمرجو الآن ان يصدر الآن النص العربي الكامل لكتاب الحيل كرادف لا بد منه للترجمة الانكليزية .

أحمد يوسف الحسن

معهد التراث العلمي العربي
جامعة حلب

المشاركون في العدد

– **عادل انبوبا** : حول تاريخ الجبر والهندسة وقد درّس تاريخ الرياضيات والعلوم العربية في الجامعة اللبنانية وفي كلية الاقتصاد الفرنسية . وتضمنت مؤلفاته دراسات حول الرياضيين المسلمين مثل الكرجي وشجاع بن اسلم وشرف الدين الطوسي والسموءل بن يحيى المغربي وغيرهم .

– **برنارد ر. غولدستاين** : درس تاريخ العلوم الدقيقة في العصور الوسطى ، من بين انجازاته القيمة اكتشافه ونشره وترجمته عن العربية ودراسته التحليلية للجزء الأكبر من فرضيات بطيلمة في الكواكب السيارة .

– **روبرت إ. هول** : اهتم بتاريخ العلوم الاسلامية وفلسفتها بشكل عام وبعلم النفس والبصريات وعلم الحركة بشكل خاص .

– **احمد يوسف الحسن** : رئيس جامعة حلب ومدير معهد التراث العلمي العربي هو مؤرخ عن التكنولوجيا عند العرب . ويقوم حالياً بنشر كتاب عن بني موسى وفن الحيل .

– **غادة الكرمي** : طبيبة ومؤرخة عن الطب العربي اهتمت بوجه خاص بالكناشات وهي كتب تطبيقية في ممارسة الطب .

– **ا. س. كندي** : ركز جهوده حول علم الفلك الإسلامي ودرس بتركيز عالٍ معظم اعمال البيروني والكاشي .

– **مصطفى موالدي** : من موظفي معهد التراث العلمي العربي كتب مقالات اقتصادية واحصائية ، ويحضر حالياً نقداً لكتاب التجريد للنسوي وهو مقدمة في الهندسة .

– **سيد حسين نصر** : مؤلفاته المدرسية العديدة والتي تنوعت مواضيعها شملت علم الأبراج والدين والتصوف والقانون وكذلك تاريخ العلوم .

– **جورج صليبا** : إنضم حديثاً الى كلية جامعة كولومبيا وشمل اهتمامه دور السوريين في نقل العلوم الاغريقية الى الاسلام .

ملاحظات لمن يرغب الكتابة في المجلة

١ـ تقديم نسختين من كل بحث أو مقال الى معهد التراث العلمي العربي طبع النص على الآلة الكاتبة مع ترك فراغ مزدوج بين الاسطر وهوامش كبيرة لأنه يمكن أن تجرى بعض التصحيحات على النص ، ومن أجل توجيه تعليمات الى عمال المطبعة . والرجاء ارسال ملخص يتراوح بين ٣٠٠ ـ ٧٠٠ كلمة باللغة الانكليزية إذا كان ذلك ممكناً وإلا باللغة العربية .

٢ ـ طبع الحواشي المتعلقة بتصنيف المؤلفات بشكل منفصل وتبعا للارقام المشار اليها في النص . مع ترك فراغ مزدوج أيضا ، وكتابة الحاشية بالتفصيل ودون أدنى اختصار .

أ ـ بالنسبة للكتب يجب أن تحتوي الحاشية على اسم المؤلف والعنوان الكامل للكتاب والناشر والمكان والتاريخ ورقم الجزء وأرقام الصفحات التي تم الاقتباس منها .

ب ـ أما بالنسبة للمجلات فيجب ذكر اسم المؤلف وعنوان المقالة بين أقواس صغيرة واسم المجلة ورقم المجلد والسنة والصفحات المقتبس منها .

ج ـ أما إذا أشير الى الكتاب أو المجلة مرة ثانية بعد الاقتباس الأول فيجب ذكر اسم المؤلف واختصار لعنوان الكتاب أو عنوان المقالة بالاضافة الى أرقام الصفحات.

أمثلـــة :

أ ـ المطهر بن طاهر المقدسي ، كتاب البدء والتاريخ ، نشر كلمان هوار . باريس ١٩٠٣ ، ج ٣ ، ص ١١ .

ب ـ عادل انبوبا ، « قضية هندسية ومهندسون في القرن الرابع الهجري ، تسبيع الدائرة » ، مجلة تاريخ العلوم العربية. مجلد ١، ١٩٧٧ ص ٧٣.

ج ـ المقدسي ، كتاب البدء والتاريخ ، ص ١١١ .
انبـــوبا ، « قضية هندسية » ، ص ٧٤ .

مصادر ودراسات في تاريخ العلوم العربية الإسلامية

سر الخليقة وصنعة الطبيعة

كتاب العلل

بلينوس الحكيم

متضمنا كتاب اللوح الزمردي لتيمسيوس الحمصي

تحقيق

أورسولا وايسر

١٩٧٩ ، ٢٧ × ٢٠ سم . ٧٠٢ صفحة النص العربي والفهرس ، ٦٦ صفحة المقدمة مع التعليق باللغة الالمانية

اشهر ما كتب في أدب السيمياء . يعتبره الاستاذ مانفرد أول ذا أهمية عظيمة كونه مفتاح أعمق أسرار الطبيعة وسيميائيي العصور الوسطى وكان مقدسا كقداسة الوصايا العشر .

يظهر اللوح الزمردي في نهاية سر الخليقة لذا توجب الاعتقاد أنه شرح للوح .

استخدمت الدكتورة وايسر في اعداد هذه الطبعة ١٧ نسخة من النسخ المعروفة وضمنت كتابها خلفية تاريخية تتناسب مع الوصف والاسلوب لكل كتاب :

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الكتاب الثاني عن الكرة الارضية والفضاء
الكتاب الثالث عن المعادن
الكتاب الرابع عن النبات
الكتاب الخامس عن الحيوان
الكتاب السادس عن الانسان

السعر ٦٠ ل.س / ١٥ دولارا . تدفع بموجب شك أو حوالة مصرفية أو نقدية للعنوان التالي :

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عاديات حلب

الكتاب الثالث
١٩٧٧

المحتوى

عادیات حلب : حولية تبحث في تاريخ الحضارة والاثار والعلوم : العدد الاول (١٩٧٥) العدد الثاني (١٩٧٦) العدد الثالث (١٩٧٧) ٦ دولارات للعدد الواحد .

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To Contributors of Articles for Publication in the Journal for the History of Arabic Science

1. Submit the manuscript in duplicate to the Institute for the History of Arabic Science. The text should be typewritten, double-spaced, allowing ample margins for possible corrections and instructions to the printer. Please include a summary in Arabic, if possible, about a third the length of the original. Otherwise let us have a summary in the language of the paper.

2. Bibliographical footnotes should be typed separately according to numbers inserted in the text. They should be double-spaced as well, and contain an unabbreviated complete citation. For books this includes author, full title (underlined), place, publisher, date, and page numbers. For journals give author, title of the article enclosed in quotation marks, journal title (underlined), volume number, year, pages. After the first quotation, if the reference is repeated, then the abbreviation *op. cit.* may be used, together with the author's name and an abbreviated form of the title.

Examples :

O. Neugebauer, *A History of Ancient Mathematical Astronomy* (New York: Springer, 1976), p. 123.

Sevim Tekeli, "Taqī al-Dīn's Method of Finding the Solar Parameters", *Necati Lugal Armagani*, 24 (1968), 707–710.

3. In the transliteration of words written in the Arabic alphabet the following system is recommended:

ʾ ,	a ,	b ,	t ,	th ,	j ,	ḥ ,	kh ,	d ,	dh ,	r ,	z ,	s ,	sh ,
ء	ا	ب	ت	ث	ج	ح	خ	د	ذ	ر	ز	س	ش

ṣ ,	ḍ ,	ṭ ,	ẓ ,	ʿ ,	gh ,	f ,	q ,	k ,	l ,	m ,	n ,	h ,	w ,	y
ص	ض	ط	ظ	ع	غ	ف	ق	ك	ل	م	ن	ه	و	ي

For short vowels, *a* for *fatḥa*, *i* for *kasra*, and *u* for the *ḍamma*.

For long vowels the following diacritical marks are drawn over the letters *ā*, *ī*, *ū*.

The diphthong *aw* is used for أو and *ay* for أي

NOTES ON CONTRIBUTORS

Adel Anbouba works on the history of algebra and geometry. He has taught the history of Arabic science and mathematics at the Lebanese University and at the French Faculty of Economics. His publications include studies on al-Karajī, Shujāʿ b. Aslam, Sharaf al-Dīn al-Ṭūsī, al-Samawʾal b. Yaḥyā al-Maghribī and other Islamic mathematicians

Bernard R. Goldstein studies the history of the medieval exact sciences. Among his notable achievements was the discovery, publication, translation from the Arabic, and analysis, of a major portion of Ptolemy's *Planetary Hypotheses*

Robert E. Hall is interested in the history of Islamic science and philosophy in general, and in psychology, optics, and mechanics in particular.

Ahmad Y. al-Hassan, Rector of Aleppo University and Director of the Institute for the History of Arabic Science, is a historian of Arabic technology. He is currently preparing a text edition of the book by the Banū Mūsā on mechanical devices.

Ghada Karmi is a physician and historian of Arabic medicine. She has been particularly interested in the *Kunnāshāt*, medical compendia used as manuals by medieval practicioners.

E. S. Kennedy has centered his efforts upon the history of Islamic astronomy, having studied intensively several of the works of al-Bīrūnī and al-Kāshī.

Mustafa Mawaldi, of the staff of the Institute for the History of Arabic Science, has written articles on economics and statistics. He is preparing a critical edition of al-Nasawī's *Kitāb al-tajrīd*, an introductory manual of geometry.

Seyyed Hossein Nasr is a scholar whose very numerous publications range over a wide gamut of subjects, including cosmology, religion, mysticism, and law, as well as the history of science.

George Saliba has recently joined the faculty of Columbia University. His interests include the rôle of Syriac in the transmission of Greek science to Islam

this field his achievements have won him well deserved fame. So far he has published English translations of the most significant Arabic books on ingenious devices, even before these treatises had been printed in the original. Naturally, any detailed work of this sort will be found to contain occasional errors. However, in view of the importance of his accomplishments, such errors are insignificant, almost negligible. By his labors Hill has paid the Banū Mūsā their full tribute. Thanks to him, the *Kitāb al-Ḥiyal* is no longer a shadowy work concerning which the non-specialist must speculate on the basis of the remarks of Ibn al-Nadīm, Ibn al-Qifṭī, and Ḥājī Khalīfa. It has taken its rightful place as a book of science and engineering, a work which we comprehend and enjoy reading. It is to be hoped that the complete Arabic text will soon appear, to complement Hill's English translation.

AHMAD Y. AL-HASSAN

University of Aleppo
Institute for the History of Arabic Science

published by him on Islamic-Arabic mechanical technology.

The distinctive feature of this English version is that it is the first in any language (including Arabic) which presents the *Kitāb al-Ḥiyal)* in its entirety. The discovery of the Topkapi MS in Istanbul has been of great value, giving Hill's work notable precedence over the German version by Hauser.

Hill's book comprises two sections, the first being the Introduction. Among other important matters dealt with in this part are: (1) the life and work of the Banū Mūsā, (2) manuscripts of the source, (3) earlier information on *The Book of Ingenious Devices*, (4) historical context, and (5) motifs.

The second part of the book contains a complete translation descriptive of the hundred devices or models which occur in the *Kitāb al-Ḥiyal*. Following each model are notes and commentary.

Hill's book ends with an appendix comprising three models whose relation to the *Kitāb al-Ḥiyal* is challengeable One occurs in the Vatican MS, another in the Topkapı version, the last in Leyden MS (Or. 168). There is a list of references consulted, and a glossary of Arabic terms and their English equivalents.

In his research, Hill depended basically on the Topkapi MS. Wherever a passage occurs in this MS, Hill relied on it primarily, comparing it to that Vatican copy. He deemed it unnecessary to collate it with the Gotha-Berlin MS. Elsewhere, the Vatican MS was taken to be the primary document, that is, in relation to those models missing in the Topkapı MS. Insofar as the last ten models are concerned, the Berlin MS is the only source available, since these are missing in both the Topkapi and the Vatican copies.

Following the translated description of each model, Hill provides a photographic reproduction of the drawing of that model as it occurs in one of the MSS. Then he displays a simplified version of the same drawing, omitting unnecessary details, such as the handles of pitchers, decorations, etc. He also puts, on these redrawn sketches, the Latin letters corresponding to the Arabic of the original. Occasionally he also provides a modern illustrative drawing, sometimes adapted from Hauser's book, with due acknowledgment. Finally, Hill inserts, following most of the drawings, appropriate remarks elucidating obscure ideas, or illuminating the fundamentals on which the particular model relies. Much repetition in these places has been saved by devoting a special section in the Introduction to an explanation of the common principles and recurrent methods used by the Banū Mūsā in designing their models.

Judging by any standards, the work undertaken by Hill is stimulating; it is to to be highly esteemed. Whoever attempts to edit a book of this nature realizes the amount of experience and the mastery of technique needed for such work. It also presupposes good knowledge of Arabic. Researches by Hill stand almost unique. For some time he has concentrated on translating and annotating works pertaining to Islamic Arabic mechanical technology, and in

The Sublime Methods of Spiritual Machines, by Taqī al-Dīn ibn Maʿrūf al-Rāṣid al-Dimashqī.[4] These three books, separated as they are by long intervals of time (respectively, the 3rd/9th, 6th/12th, and the 10th/16th centuries) constitute three major links in the chain of mechanical engineering achievements, a component of Islamic-Arabic civilization. It is to be hoped that the recovery and publication of other books will supply the missing links to the chain.[5]

Thus the Islamic-Arabic legacy in the field of ingenious devices begins with the work of the Banū Mūsā, a book which won resounding fame in the Arabic literature. Fortunately, this is one of the few books by the Banū Mūsā that have survived. However, in spite of its being widely known, the extant MSS are few. Today there are only three major copies: Topkapi Saray Ahmet III 3474; Vatican MS 317; and a third MS, divided between the Gotha library (No. 1349) and Berlin (No. 5562). The Topkapi MS has only recently come to light.[6]

It was towards the end of the last century that historians of science began to devote their attention to the *Kitāb al-Ḥiyal* by the Banū Mūsā. Serious studies on this book, however, were not conducted before the first decades of this century, when Wiedemann and Hauser published articles on the drinking pitchers, and described figures 85-87 of the *Kitāb al-Ḥiyal.*[7] Hauser later published a lengthy book into which he incorporated the remaining figures.[8] Thus the work became available in German. Wiedemann and Hauser depended primarily upon the Vatican MS, and, in a secondary sense, upon the Gotha-Berlin version. Because the texts in these MSS were sadly truncated and seriously defective, Hauser exerted much effort in attempting to interpret the figures. In consequence he had to take liberties with the translation, recasting the German in such manner as to render the text intelligible from the technical point of view.

The latest and most important research on the *Kitāb al-Ḥiyal* is the book here reviewed, the English translation of the complete text by Donald Hill. He thus continues an important project commenced in 1973 with his English translation of the book of al-Jazarī.[9] This is in addition to other research

4. The Arabic text has been edited by Ahmad Y. al-Hassan, *Al-Ṭuruq al-saniya fī al-ālāt al-rūḥāniya* (Aleppo, *IHAS*, 1976).

5. For word of an additional link, see Donald R. Hill, "A Treatise on Machines...", *Journal for the History of Arabic Science*, 1 (1977), 33-46.

6. See the review of Hill's al-Jazarī translation by David A. King, "Medieval Mechanical Devices", *History of Science*, 13 (1957) 284-289.

7. Eilhard Wiedemann and F. Hauser, "Über Trinkgefässe und Tafelaufsätze nach al-Gazarī und den Banū Mūsā", *Der Islam*, 8 (1918), 55-93, 268-291.

8. F. Hauser, "Über das Kitāb al-Ḥiyal. . .", *Abhandl. zur Gesch. der Naturwissenschaften und der Medizin* (Erlangen, 1922).

9. See Note 3 above.

Book Review

Donald R. Hill (Translator). *The Book of Ingenious Devices* (Kitāb al-Ḥiyal) *by the Banū (sons of) Mūsā bin Shākir*. Translated and annotated by Donald R. Hill. Dordrecht, Holland: D. Reidel Publishing Co., 1979. x + 267 pages. Dfl. 130 / $63.

The Banū Mūsā lived in the 3rd (H.)/9th (A.D.) century, when Arabic civilization had reached its zenith. In the reign of al-Ma'mūn and the caliphs who succeeded him, the three sons of Mūsā bin Shākir-Muḥammad, Aḥmad, and al-Ḥasan - played a prominent part in promoting the sciences, particularly mathematics, astronomy, and mechanics. This they did through their writings, as well as by their pervasive influence on the translation movement from Greek into Arabic. But while the writings of the Banū Mūsā were voluminous and varied, the work which stands out as distinctive is *The Book of Ingenious Devices (Kitāb al-Ḥiyal)*. Wherever mention is made of the Banū Mūsā, this ingenious piece of work stands as their greatest achievement.

In the *Mafātīḥ al-ᶜUlūm*,[1] al-Khwārizmī sets down *al-ḥiyal* (the science of ingenious devices) as one of eight fundamental disciplines. He then divides it into two parts: one pertains to the moving of weights by application of mechanical advantage; the other deals with ingenious devices for moving water, and the making of curious vessels, along with the related art of automata.

In later classifications of the sciences, *al-ḥiyal* found itself categorized as a branch of *al-handasa*, not in the mathematical sense (geometry), but rather in the technological (the engineering)[2] sense.

In any case, and apart from classifications of the sciences, so widely different from age to age, the science of ingenious devices, or *al-ḥiyal*, falls within the scope of mechanical engineering, as it deals with machines, instruments, and hydraulic and mechanical equipment.

Until recently, only two Arabic books on the subject had been widely known, one, *The Book of Ingenious Devices* by the Banū Mūsā, the other *A Compendium on the Theory and Practice of Ingenious Devices* by Badīᶜ al-Zamān ibn al-Razzāz al-Jazarī.[3] To these two have now been added a third,

1. Muḥammad b. Aḥmad b. Yūsuf al-Khwārizmī, *Mafātīḥ al-ᶜUlūm* (Cairo, Idārat al-Ṭibāᶜat al-Munīriya, 1342 H.), p. 191.

2. Aḥmad al-Qalqashandī, *Ṣubḥ al-Iᶜshā* (Cairo, al-Maṭbaᶜat al-Amīriya, 1913), vol. 1, p. 476.

3. The Arabic text has recently been published *Al-Jāmiᶜ bayn al-ᶜilm w'al-ᶜamal al-nāfiᶜ fī ṣināᶜat al-ḥiyal*, by Ibn al-Razzāz al-Jazarī, edited by Aḥmad Y. al-Hassan, Institute for the History of Arabic Science, hereafter *IHAS*, (Aleppo, 1979). This was preceded by the English translation *The Book of Knowledge of Ingenious Mechanical Devices* by Ibn al-Razzāz al-Jazarī, translated and annotated by Donald Hill (Dordrecht, Reidel, 1973).

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Dr. A. Y. al-Hassan

Professor al-Haschmi Honored

On June 7, 1979, the Institute for the History of Arabic Science held a ceremony in honour of Professor Muhammad Yahya al-Haschimi. The program included a poem in praise of this noted historian of science by ʿOmar Abu Qaws. Drs. Taha Ishaq Kayali, ʿAbd al-Salam ʿUjeili and Mr. Fuad Aintabi delivered speeches dealing with Professor Haschmi's life and work. His books were on display during the ceremony, and he was nominated for the Syrian Order of Merit. His book on plants, which Dr. Nazir Sankari is presently revising, will be published in the near future.

Professor Haschmi was born in Aleppo in 1904. He studied chemistry and philosophy in Germany, and took the doctorate for his study on al-Bīrūnī's *Kitab al-Aḥjār*. During his long professional career he has taught in secondary schools in Aleppo, and lectured at the university. His special interest has been the history of Arabic and Islamic science and its transmission to the West. He has published a number of articles and several books. His most important achievement was the establishment of the Syrian Society for Scientific Research in 1957. Professor Haschmi has retired from teaching, but is still pursuing his scientific activities, in addition to being an active member of the Syrian Society for the History of Science.

Professor Sezgin Winner of King Faisal International Award

Professor Fuat Sezgin, the candidate of the Institute for the History of Arabic Science for the King Faisal International Award, won the award for Islamic studies, bestowed in recognition of his six volume work, *Geschichte des arabischen Schrifttums.* Professor Sezgin has already spent twenty years collecting and compiling his source material for this monumental publication, and two further volumes are still in preparation.

One of the orientalists present at the congress held in Würzburg (FRG) in 1968, said in praise of this great achievement, "Brockelmann was the centre of attention for many a year, but the *Geschichte des arabischen Schrifttums* will become one of the 20th century's most important contributions to Arabic literary culture and the classification of the immense Arabic heritage".

In the introduction to the first volume of the Arabic translation from its German original, Dr. Fahmi abu al-Fadl says, "This is not only a book on science, but also a proof of great achievement if we consider that such a work is usually compiled by a group of scholars. Yet the *Geschichte des arabischen Schrifttums* is entirely the work of Professor Sezgin".

Professor Sezgin was born in Istanbul in 1924 where he studied, taking his doctorate in Islamic Science and Persian Studies. For a number of years he was on the faculty of the University of Istanbul.

The History of Rhetoric written in Turkish (1947), and *Studies on Bukhari's Compendium of Sources*, published in 1956, are only two of Professor Sezgin's numerous works. In 1960 he moved to Germany, where he lectured for two years at the Institute for Semitic Languages of the University of Frankfurt. Subsequently he was named a visiting professor at the Institute for the History of Natural Science at the same institution. He then obtained a chair at the University of Frankfurt with all the rights of a German professor, although he retained his Turkish nationality.

He was granted the award on February 27, 1979, in an official ceremony held in Riyad under the patronage of His Majesty King Khaled ibn Abdul Aziz. Dr. Ahmad Y. al-Hasan, Rector of the University of Aleppo and Director, Institute for the History of Arabic Science, attended the official celebration.

The award consists of a certificate bearing the name of the prize, a valuable medal, and a sum of money equivalent to 200,000 Saudi Rials.

The Second International Symposium for the History of Arabic Science

The opening ceremonies were convened on Thursday, 5 April, 1979, before an audience of seven hundred people. The scientific meetings commenced on the same day. These continued through Monday, 9 April, being held in various auditoriums of the University of Aleppo.

The meetings included three seminars, having the following themes:

The Place of Science and Medicine in Medieval Islamic Civilization
The History of Algebra
and *The Transmission of Arabic Science to the Latin West*

Each seminar was addressed by a group of from two to four invited speakers, after which the meeting was thrown open for general discussion.

In addition to the seminars, there was a total of seventeen sessions for the presentation of some 114 short papers on topics chosen by the participants, opportunity being given for questions and remarks from the floor after each paper. These sessions were organized by fields of study, with two or more running simultaneously. The history of medicine was by far the most popular subject with thirty-four papers. Next was mathematics with eighteen, thence lesser numbers of presentations involving astronomy, the earth sciences, technology, astrology, alchemy, physics, agriculture, and so on.

Interspersed with the scientific sessions were lectures and film showings of general interest, notably of the excavations at the famous nearby site of ancient Ebla.

The Institute for the History of Arabic Science prepared exhibits of publications of the University of Aleppo and of the numerous objects which make up the first acquisitions for the Institute's future history of science and technology museum.

The last day of the Symposium concluded with a general meeting of participants, for the adoption of resolutions, and a final banquet.

On Tuesday, 10 April, those of the departing visitors who so chose were escorted on tours to Ebla and the Krak des Chevaliers via Homs, or to Lattakia and Ugarit.

Scholars resident in a total of twenty-seven different countries were present. Naturally, the twenty-three from Syria, the host country, made up the largest group. There were twenty-seven from the other Arab countries, about a third of these being from neighboring Iraq. The eleven participants from the USSR made up the largest single delegation, with West Germany, France, the U. S. A., and the United Kingdom not far behind

Many of the participants expressed gratification at the level of the material presented, and with the arrangements in general. The organizers of the Symposium may congratulate themselves upon a job well done.

by Sbath in his published catalogue, is not noted by either Brockelmann, Sezgin, or Ullmann in their bibliographies.[9]

The Manuscript

(Number: Antaki 1)

Damaged and faded leather cover. The binding has come apart and most of the pages are loose, but otherwise the manuscript is well-preserved. Stained pages. Almost no marginal notes. The first page contains one owner's seal and four entries in different hands. One of these, which appears to be more recent than the text, reads:

كتاب المنصوري في حفظ الصحة ومعالجة الامراض لمن يحضره الطبيب تأليف الشيخ الحكيم ابي بكر محمد بن زكريا الرازي

182 ff. Complete. Paginated in ink. Ends on p. 364.
18.5 × 11.5 cm. 22 lines.
Legible naskhi script, partly vocalised. Red ink headings. No scribe's name.
Undated. Probably 7th/13th century (as Sbath).

Begins:

بسم الله الرحمن الرحيم رب يسر وأعن برحمتك مجدا كتب لله [sic] محمد بن زكريا الرازي للمنصور
بن اسحاق اسمعيل بن أحمد فقال اني جامع للامير أطال الله بقاه جملا وجوامع ونكتا وعيونا من صناعة
الطب وتحدد في ذلك الاختصار والايجاز وذاكر من ما لا يحدث ..

Ends:

فليوخذ لحم رطل من وزن درهم مصطكي ودرهم قرنفل ودرهم سنبل قصير في خرقة ويلقى عند الطبخ فيه
ان شاء الله تعالى واذ اتينا على جميع المقالات والفصول المذكورة في صدر هذا الكتاب فقد كمل كتاب عد
ولله المعين والموفق للصواب وهو حسبي ونعم الوكيل ولا حول ولا قوة الا بالله العلي العظيم تم الكتاب والحمد
لله حق حمده وصلى الله على سيدنا محمد وآله وصحبه وسلم تسليما

It is of course always useful to discover the whereabouts of an Arabic scientific manuscript. But it is particularly useful in this case for two reasons: firstly, there is no modern printed edition of *K. al-Manṣūrī*, and secondly, the book is of great importance to the history of Arabic medicine and mediaeval learning. In addition, this manuscript is especially valuable because it is complete, well-preserved, and appears to be early. Many of the surviving manuscripts of *K. al-Manṣūrī* are incomplete, sometimes lacking as much as a half or a third of the original text.

It is fortunate that this manuscript has been released from private ownership and is now available for scholarly use.[10]

9 C. Brockelmann, *Geschichte der arabischen Litteratur* (Weimar: Felber, 1898-1902), Vol.I, pp. 233-5 (one would not of course expect the Sbath manuscript to be mentioned in this edition), *Supplement*, (Leiden: Brill, 1937-42), Vol. I, p. 417. Sezgin, *op. cit.*, Vol III, pp. 281-2; M. Ullmann, *Die Medizin im Islam* (Leiden: Brill, 1970), p. 132.

10. In this connection, it should be mentioned that I am currently preparing an edition of Book 9 for publication by the *IHAS*. This MS will be one of those used in the preparation of this edition.

scripts of the work extant, dispersed in various eastern and western libraries. This large number and the wide temporal span of the surviving manuscripts is further testimony to its popularity.[6] Yet, apart from Reiske's Arabic and Latin edition of 1776, there has been no Arabic edition of the work in modern times. The first *maqāla* was edited and translated into French by de Koning in the early part of this century.[7]

K. al-Manṣūrī is moderately large, (the manuscript length averages at 200 ff.). It deals with all the major topics of medical importance of the time, as the subjects of the ten *maqālāt* indicate:

The Form and Appearance of Organs
Knowledge of the Temperaments of Bodies and the Preponderant Humours in Them
The Faculties of Foods and Medicines
The Preservation of Health
Cosmetics and External Diseases
The Management of the Traveller
Bonesetting, Wounds and Ulcers
Poisons and Insect Bites
The Diseases from Head to Toe
Fevers, Coction, Crisis, the Urine and the Pulse

In 1977, the Institute for the History of Arabic Science at Aleppo received a gift of 255 manuscripts from a well-known art collector of Aleppo, Mr George Antaki. Among the medical works was a manuscript of *K. al-Manṣūrī*. The only copy of this book previously known to have been in Aleppo was the one mentioned by Father Paul Sbath in his 3-volume catalogue of the manuscripts held in private collections in Aleppo. Here, he refers to a manuscript of *K. al-Manṣūrī* in the collection of the consul for Holland, M. Rodolphe Poché. The entry in Sbath's book is characteristically brief and gives no description of the manuscript.[8] Careful inquiry has established that this manuscript of the Dutch consul is the same as that donated to the *IHAS*. It had passed from that owner into the posession of others and thence to the final purchaser who donated it to the *IHAS*. With the manuscripts came also a hand-written list of their titles, authors, and brief descriptions. This is contained in a small exercise book, said to have been written by Paul Sbath in the 1930s in preparation for a fuller catalogue (which he never undertook). The entry for *K. al-Manṣūrī* dates it as 13th century (A. D.) and marks it as 'précieux'. No other description is given. The existence of this manuscript, although it was listed

6. There are manuscripts of this work dating from the 5th/11th century until the 12th/18th century. For details, see F. Sezgin, *Geschichte des arabischen Schrifttums* (Leiden: Brill, 1967), Vol. III, pp 281-2.

7. P. de Koning, *Trois traités d'anatomie arabe* (Leiden: Brill, 1903), pp. 2-98.

8. P. Sbath, *al-Fihrist, catalogue des manuscrits arabes*, 3 volumes plus supplement, Cairo, 1938-40, Vol.I, p.99. Elsewhere (Introduction, p.vii), Sbath says that this Consul had a large collection of Arabic manuscripts.

NOTES AND CORRESPONDENCE

Notice of Another Manuscript of al-Rāzī's Kitāb al-Manṣūrī

GHADA KARMI*

ONE OF THE MOST FAMOUS of Abū Bakr Muḥammad b. Zakariyya al-Rāzī's books was the comprehensive book on medicine which he dedicated to the Samanid prince, Abū Ṣāliḥ al-Manṣūr b. Isḥāq, (after whom it was known as the *K. al-Manṣūrī*). It was an extremely celebrated work in the Latin West throughout the Middle Ages, and was translated into Hebrew, Greek and Latin, the last by Gerard of Cremona in 1175.[1] It was printed in Latin in 1481, and was reprinted many times thereafter. There are also many Latin manuscripts of the book extant, further proof of its popularity in the West.[2] *K. al-Manṣūrī* is divided into ten treatises, or *maqālāt*. The 9th *maqālā*, or *Liber Nonus* (alternatively known as the *Nonus Almansoris*), which deals with the diseases from head to toe, became especially important in Latin translation. It was printed many times, particularly in the 15th century, and was extensively used and commented on in the 15th and 16th centuries.[3] The most famous of these commentaries was Andreas Vesalius' paraphrase, which was published in 1537.[4]

K. al-Manṣūrī was also popular and important in the Arabic East. Abū'l-ᶜAbbās al-Majūsī, the 10th-century author of the medical encyclopaedia, *Kāmil al-Ṣināᶜa*, says in his introduction that al-Rāzī had surpassed all others in the excellence of his book, *K. al-Manṣūrī*.[5] Today, there are no less than 47 manu-

*The Wellcome Institute for the History of Medicine, 183 Euston Road, London, N.W 1, U. K.

1. See M. Steinschneider, *Die europäischen Übersetzungen aus dem Arabischen bis Mitte des 17. Jahrhunderts* (Vienna, 1904-5), p.25.

2. See L. Thorndyke and P. Kybre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, revised and augmented edition, (Mediaeval Academy of America, 1963), pp. 272, 471, 1053, 1375, 1538, also, S. Punsier, "Catalogues des manuscrits medicaux des bibliothèques de France", *Sudhoffs Archiv*, 2(1908), 36-7.

3. See H. Schipperges, "Bemerkungen zu Rhazes und seinem Liber Nonus", *Sudhoffs Archiv*, 47(1963), 373-7.

4. Andreas Vesalius, *Paraphrasis in Nonum Librum Rhazae* (Basle, 1537).

5. Al-Majūsī, *Kāmil al-ṣināᶜa* (Cairo, Bulaq, 1294/1877), Vol. I, p. 5, l. 1-3

En collaboration avec René R. J. Rohr, "Deux astrolabes-quadrants turcs", *Centaurus*, 19(1975), 108-124.

1976

"Un cadran solaire juif", *Centaurus*, 19(1976), 264-272.

"Un compendium de poche par Humphrey Cole (1557)", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, 1 (1976), 1-11.

1977

"Quelques aspects récents de la gnomonique tunisienne", *Revue de l'Occident Musulman et de la Méditerranée*, (Aix-en-Provence, France), 1977, 207-221.

"Un cadran de hauteur", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, 2 (1977), 21-25.

En collaboration avec D. A. King: "Ibn al-Shāṭir's Ṣandūq al-Yawāqīt: An Astronomical Compendium", *Journal for the History of Arabic Science*, 1(1977), 187-256.

1978

"Un cadran solaire grec à Aï Khanoum, Afghanistan", *L'Astronomie* (Paris), 92 (1978), 357-362.

En collaboration avec D. A. King: "Le cadran solaire de la Mosquée d'Ibn Ṭūlūn au Caire", *Journal for the History of Arabic Science*, 2(1978), 331-357.

"Un texte d'ar-Rudani sur l'astrolabe sphérique", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, 3(1978), 71-75.

1979

"Astrolabe et cadran solaire en projection stéréographique horizontale", *Centaurus*, 22(1979), 298-314.

tan. Toutes ses recherches furent brutalement interrompues par son décès en décembre 1978, alors que deux articles étaient encore sous presse.

On trouvera ci-après une liste de ses publications.

1969

"L'histoire du cadran solaire", *La Suisse Horlogère*, (1969), 93-101.

1970

"Note sur le cadran solaire de Brou", *L'Astronomie*, Paris (1970), 83-85.

"Les cadrans solaires polyédriques du musée du Pays Vaurais", *Bulletin de la Société des Sciences, Arts et Belles-Lettres du Tarn*, N. S., 29(1970), 357-365.

1971

"Les méridiennes du château de Versailles", *Revue de l'Histoire de Versailles*, 59(1971).

"Un cadran solaire astronomique", *L'Astronomie*, Paris (1971), 251-259.

1972

"Le cadran polyédrique du cloître de Brou", *Bulletin de la Société des Naturalistes et Archéologues de l'Ain*, Bourg-en-Bresse, France, (1972), no. 86, 77-82.

"Le cadran aux étoiles", *Orion*, (Schaffhouse, Suisse), 30(1972), 171-175.

"Un cadran solaire de hauteur", *Sefunim* IV, Bulletin 1972-1975, (Haifa), 60-63.

"Le cadran solaire de la mosquée Umayyade à Damas", *Centaurus*, 16 (1972), 285-298, reproduit dans E. S. Kennedy, and I. Ghanem, eds., *The Life and Work of Ibn al-Shāṭir: an Arab Astronomer of the Fourteenth Century*, (Alep: Institute for the History of Arabic Science, 1976), pp. 107-121.

1973

"Le monument solaire de Bagneux", *Histoire Archéologique*, Bulletin de l'Association des Amis de Bagneux, (Bagneux, France), 1973, 521-529.

1974

"Le cadran solaire multiface de l'Abbaye Sainte-Croix de Bordeaux", *Revue Historique de Bordeaux et du département de la Gironde*, (France), 1974, 31-41.

"Le cadran solaire analématique, histoire et développement", *Centre Technique de l'Industrie Horlogère*, (Besançon, France), no. 74, 2057, 1974, 1-37. Il existe une traduction allemande due à René R. J. Rohr parue dans *Uhren Technik* (U. T.), 2 (1974), 1-15.

"Le cadran lunaire", *Orion*, (Schaffhouse, Suisse), 32(1974), 3-11.

1975

"Un cadran solaire oublié", *Orion*, (Schaffhouse, Suisse), 33(1975), 179-182.

Éloge

LOUIS JANIN

17 OCTOBRE, 1897 - 29 DÉCEMBRE, 1978

Par C. Nallet, René R. J. Rohr, et D. A. King*

DIPLOMÉ des Hautes Etudes Commerciales, Docteur en Droit, M. Louis Janin a fait toute sa carrière dans le commerce international en tant que Directeur d'une grande banque parisienne. Il a eu six enfants, dix-huit petits-enfants et de nombreux amis.

Au cours de sa vie professionnelle, M. Louis Janin a travaillé en Algérie et a eu de nombreux contacts avec les pays arabes.

Ce n'est qu'après avoir pris sa retraite, en 1965, à l'âge de 68 ans, qu'il s'est intéressé à la gnomonique, et c'est à partir de cette date là qu'il lui a consacré son temps et ses efforts. Son intérêt pour la gnomonique arabe remonte à sa découverte de l'absence de publication sur le splendide cadran de la Mosquée Omayyade à Damas. Par la suite, il a visité le Caire pour examiner tous les cadrans médiévaux que l'on peut y trouver, et il y a appris que le plus splendide de ceux que l'on connaît était celui de la Mosquée d'Ibn Ṭūlūn qui n'existe plus que dans une reproduction fidèle préparée par les savants qui accompagnaient Bonaparte en Egypte. Une de ses publications les plus récentes traite d'un cadran extraordinaire, d'origine grecque, qui a été découvert en Afghanis-

*12, avenue Carnot, 75017 Paris, France.

primarily of doctrinal responses; and when philosophy became focused upon illuminationist metaphysics, the desirability of natural philosophy and the mathematical sciences of nature was seriously reduced.[45] But Avicenna's thought, as the foremost exemplar of *falsafa*, would also have played the leading part on the side of the ancient disciplines in the general 'religious dialogue' that I have posited. If the description is essentially correct this far, then one may affirm with confidence that Ibn Sīnā's theoretical psychology exercised a decisive influence upon the history of the Greek sciences and upon the evolution of Islamic cultural history in general. This is the conclusion that I have been the most anxious to substantiate; but even in a longish paper adequate support can be produced for only a part of the necessary argument. I hope, though, that I have treated primarily those points which have most significance for the broader issues.

Let me add a final observation. If this historical assessment has been an accurate one, then the career of philosophy and of all the Greek sciences was pressed forward in the very centre of Muslim culture, not at the periphery as is often supposed. Indeed philosophy and her sister disciplines will need to be regarded as having developed in ways and by processes which were common to most fields of intellectual endeavour in the Islamic middle ages and which seem to have been among the most characteristic and important features of Muslim civilization.

45. All the natural sciences were harmed in this way, including psychology itself. But psychological theory escaped to a considerable extent, because it was able to direct its inquiries towards the ontology of intellects and related subjects; having thus been itself transformed, it came to occupy a position midway between 'physics' and metaphysics.

of the acquisition of knowledge which left him in exactly the moderate illuminationist posture he wanted. The analysis of 'experience' *(tajriba)* in the *Burhān* was a crucial step in securing his epistemological doctrines. *Tajriba* could be useful, but it had a strictly limited rôle, and there was no instance in which it could not, in the end, be avoided. 'Experience' belonged to the estimative faculty, 'knowledge' to the intellect; and the active principle of intellection resided in a celestial being. Avicenna's establishing of this external-intellection theory of knowledge was what was most decisive, I have argued, in determining the subsequent relationship of the Greek sciences and their proponents to the followers of the other ways of Muslim knowledge.

Ibn Sīnā's attitude to empirical knowledge I have mainly attributed to an ultimately Muslim belief in personal salvation. To this, too, I have credited his interpretation of the ensoulment of the human embryo presented in the *Kitāb al-Hayawān*. Finally, since paradise was to offer eternal intellection as its highest reward, it was in this connection also that the chief ontological problems were generated. Avicenna took various measures to reconcile actual intellection with incorporeal individualization, but he had no real success.

The discussions in this paper were designed to show how very much of the philosophy of Ibn Sīnā was connected to his psychology and how crucially his system depended upon the elaboration of a consistent general theory of the soul. In the Islamic intellectual world of the late ninth, tenth, and early eleventh centuries, moreover, it is correct to say that a very large proportion of the leading issues lay within psychological theory or derived immediately from doctrines there - an assertion for which my earlier list will have to provide a sufficient *prima facie* case. In passing, without proof, I have offered an analysis of the development of Islamic intellectual culture wherein the fundamental process is seen as a 'dialogue' among the several groupings of Muslim thinkers, one of which comprised the *falāsifa* and other adherents of the Greek sciences. Conceived here as basically a religious debate, it had as its underlying question the sort of *ʿilm* that was to be accepted as true and was thus to supply the proper understanding of the religion. The full examination of *tajriba* and related matters above was intended in part to clarify this picture of a general debate and to present a notable example of what I conclude was involved in it. If this interpretation represents the historical situation properly, then the further statement may be made, that through the medium of this 'dialogue' psychological theory exerted a major force in the final shaping of medieval Islamic culture.

Probably no one doubts that Ibn Sīnā was a key figure in the history of Muslim thought. The real task is to learn by what means Avicenna's philosophy came to change the course of the Greek sciences in Islam and thus to alter the development of Muslim culture as a whole. A tentative answer is now available. The transformation in *falsafa* itself was relatively direct, a matter

But if Ibn Sīnā was not mystical in his outlook, neither was he empirical. He had arrived at a position where he hoped to have the best of both worlds. The practical result, in fact, was almost to gain neither. The salvaging of his philosophy did not begin until two centuries later, and then only in the Iranian schools; there it was made something wholly mystical, with logic and the sciences as pure propaedeutic.

Avicenna's illuminationism rendered *tajriba* superfluous and left *falsafa* impotent to serve as a basis for the progressive investigation of nature. The epistemological foundation of philosophy was made exactly the same as that of the traditional religious sciences in Avicenna's system, *viz.*, revelation from the Active Intellect; but of course philosophy could be given neither the direct authority of a God-sent Message nor the social support available to the Qur'ānic disciplines or even to *kalām*. Yet the illumination accessible to the philosopher had little of the bliss and ecstasy of the union with God claimed by the *ṣūfī*'s. Without saving the sciences of nature, without gaining the felicity of the mystics, and without capturing any of the social might or religious authority of the jurists or even the lesser strength of the theologians, Ibn Sīnā failed his side badly in the general Islamic cultural debate over the nature of proper Muslim ʿ*ilm*.

His was, to be sure, an extraordinarily difficult task, and one cannot deny the brilliance, or at the very least the thoroughness and competence, of the philosophical synthesis achieved by Avicenna. He provided solutions to the primary, psychological problems with which he had been faced, even if he left unanswered a set of derivative questions in ontology. Using an exceptionally oblique method of presentation Ibn Sīnā produced a non-Aristotelian account

esp. to the notes), pp. 129-131, and ch. 5 (pp 145-196: published separately as *La Connaissance mystique...*, also cited in note 34). Gardet and Massignon had viewed Avicenna's system as ultimately 'mystical', whether Plotinian or *ṣūfī*; but in the next year appeared Shlomo Pines's 'La "Philosophie orientale" d'Avicenne et sa polémique contre les Bagdadiens', *Archives d'histoire doctrinale et littéraire du moyen-âge*, 27 (1952), 5-37, which found Avicennianism to be an esoteric Peripateticism of essentially the sort that has been described in this paper. A rejoinder came from Henry Corbin, in his *Avicenne et le récit visionnaire* (2d ed., Paris, 1954), Eng tr by W Trask as *Avicenna and the Visionary Recital* (New York, 1960), pp. 271-278, there he examined both the article by Pines and one by Massignon ('La Philosophie orientale d'Ibn Sīnā et son alphabet philosophique', pp. 1-18 in *Mémorial Avicenne, IV: Miscellanea* (Cairo, 1954)) before pressing an even more mystical reading than that of Massignon. Pines's 'La Conception de la conscience de soi . . .' (see note 34 above) is also pertinent here; and *cf.*, finally, Louis Gardet and M M Anawati, *Mystique Musulmane* (2d ed., Paris, 1960), *passim*.

The present paper has relevance to the problem of mysticism in Ibn Sīnā only if Pines's side of the argument is largely correct I am gratified in this respect by the support received from the conclusions of the careful investigator of *Al-Risāla al-Aḍḥawiyya*, Francesca Lucchetta, who in her introduction to that work (*ed. cit*, note 4 above, pp. lv-lvi) says she has found no evidence for *ittiḥād* of any kind, for *ittiṣāl* with entities other than the Active Intellect, or for direct contemplation, intellectual or otherwise, of the One Avicenna's philosophy seems to have held within it only that moderate illuminationism of an intellective kind which has been presented above.

But such knowledge is still intelligible, not more, and is logically – syllogistically – ordered, although all interrelationships among the intelligibles are known at the same time. I have not found any passage in the *Shifā'*, nor in the *Ishārāt* the *Aḍḥawiyya*, nor elsewhere, that suggests the least possibility that a human soul or intellect, in this life or the next, may become conjoined to a being higher than the lowest of the celestial intellects, or, in other words, that it may participate in any knowledge or mode of being higher than that of the Active Intellect. Ibn Sīnā's whole cosmological system demonstrates, and is in part designed in order to demonstrate, the impossibility of the uniting or conjoining with the One by any finite being, even the highest celestial intellect.[43] Moreover, far from proving mystical *fanā'* (extinction of self) in the state of 'conjunction', Avicenna commits his efforts to preserving individual identity there.

One may talk of more than one sort of spiritual union: the mystic may say 'I am God', in which case there is *ittiḥād* ('unification') unqualified, and the identity of the mystic has been lost in that of God himself (or of the One); or he may claim that he is within or joined to God, a qualified *ittiḥād*, wherein his identity is retained; a thinker of Avicenna's persuasions, however, may only assert that he is within or joined to a celestial intellect, where his condition is the intellective *ittiṣāl* that has been discussed. This state is nothing like the 'light' *(nūr)* or the 'tasting' *(dhawq)* described by the *ṣūfī*'s nor the kind of union with the One that Plotinus claimed to achieve. In the *Ishārāt* and other places Ibn Sīnā makes use of the language of the *ṣūfī*'s; but it is not *their* doctrines he proceeds to expound. So, although he is unquestionably an illuminationist in a certain precisely restricted sense, Avicenna cannot be considered a mystic of either a *ṣūfī* or a Plotinian kind. Nor, it seems to me, can he be termed a mystic in any other significant way.[44]

pp. 190-209 and 214-215, *ed. cit.* in note 4.

The notion of eternal, or prophetic, intellection as timeless, or simultaneous, syllogizing is reasonably clearly expressed in the *Kitāb al-Nafs* passage (Eng. tr., p. 36 in *Avicenna's Psychology*, cited above in note 10, in the Arabic text of the *Najāt*, p. 167, *ed. cit.* in the same note); timeless syllogizing seems also to be the activity that Aristotle attributed to the Prime Mover at the end of Metaphysics XII: 9.

43. Avicenna's cosmology is fully expounded in the *Shifā'*. See *al-Ilāhiyyāt* VIII: 7 and IX: 2, 3, and 4, esp. IX: 3 and 4, *cf.* Nasr, *Introduction ..* (cited in note 35 above), pp. 202-207 – but use this account with care, for much of it is based on a *risāla* that is almost certainly spurious.

44. Ibn Sīnā's 'mysticism' and the associated issue of his 'esoteric philosophy' have been taken up by nearly every Avicennian scholar active since World War II, and for good reason, as the solutions to these problems must form a fundamental part of any general interpretation of Avicenna's thought. I shall mention here only enough writings to furnish the basic information and make clear the main points of view. The state of opinion at the end of 1950 was compactly exhibited in the special issue of *La Revue du Caire* for June, 1951, which had articles on the subject by Louis Massignon and (two) by Louis Gardet, plus two more on related topics by Aḥmad Fu'ād al-Ahwānī and M. M. (G. C.) Anawati. Gardet's views were more fully explained in his *La Pensée religieuse* (cited in note 34 above) published the same year; see pp. 23-29 (which follows the text of the first Cairo article, but with additions,

a potential intellect, how can it be genuinely a component of the soul, which ought to subsist at a different ontological level? If, again, it really is an intellect, how can it have been individuated for a given matter *(i.e.,* its body) in the first place? Why should the rational soul (whether truly soul or truly intellect) be supposed able to return to the *dator formarum* as something self-subsistent when it has left it fit only to be joined in a necessary connexion to one particular body? What, principally and finally, is a 'soul' doing in a celestial intellect *qua* soul, or, if it is there as an intellectual entity, how can it have retained its individuality? Ibn Sīnā by no means avoids these questions, but he does not answer them cogently. He seems to exploit the intrinsic ambiguity of designations such as 'rational soul' and 'individual intellect', using them equivocally in senses that are in fact incompatible.

This is but one place, although a principal one, to be sure, where the philosophical structure erected by Ibn Sīnā reveals large cracks in its fabric. The source of many of them lies in psychological theory, as has already been said, but of these, most emerge to view only as flaws in his ontology. The ontological problems that are created by basic psychological doctrines like the external-intellection theory of *ʿilm* often may be traced further back, to the basic Muslim belief in individual immortality, in particular. Many of the difficulties in the ontology of Avicenna he himself fails to isolate, and not a few he covers over; they remain largely unsolved. Like the intricate Christological enigmas of Patristic theology they are most obvious in what may be called, rather pedantically, ontological anthropology. These faults in the ontology of Ibn Sīnā must be ranked among the intrinsically most damaging in his entire system; some of them, moreover, relate to doctrines meant to replace traditionally interpreted Qur'ānic dogmas. It is not surprising that the ontological failings as a group become particularly injurious to the reputation of Avicenna's philosophy in the Islamic world.

Since relevant information is now to hand, it is perhaps excusable to turn briefly aside to the issue of Ibn Sīnā's mysticism. Certainly it is the teaching of Avicenna that authentic knowledge comes to the human mind only through conjunction *(ittiṣāl)* with a celestial intellect. Prophets, and some others, at least in their moments of greatest insight, grasp the whole or most of the intelligible world of the Active Intellect simultaneously; and continuous, or rather timeless, existence in this condition of complete understanding is the highest felicity in Avicenna's paradise. This state of being is an immaterial and eternal possession of conscious, self-aware, and necessarily actual knowledge of the sort just described.[42]

42. The main theoretical treatments of the higher human intellectual (or psychical) states are *Al-Shifā', Kitāb al-Nafs*, V 6, esp. pp. 249-250, *ed. cit.* in note 8 above, and *al-Ilāhiyyāt* IX 7, pp. 425-426 and 429 (in vol. II), *ed. cit.* in note 13, and *Al-Risāla al-Aḍḥawiyya*, ch. 7, *passim*, esp.

individuation and has indeed evolved the fundamentals of an ego-doctrine.[39] His 'ego', I believe, may be described by the formula 'individual rational soul' = 'intellect' + 'ego'; that is to say, the rational soul comprises an intellective faculty and an unanalysed immaterial principle of individuation, which may be called an 'ego' (literally, and in the standard philosophical sense, if not precisely in any of the technical meanings of the term in modern psychology). It also seems that the potential intellect, in the narrowest sense, does, when actualized, become identical to the intelligibles which the rational soul is 'borrowing'. But even so little as this is never made explicit. Some negative conclusions may be confidently drawn, however: that there is little real influence here from the Plotinian 'we', or the Stoic 'attention', and none from an 'attentive' *(prosektikon)* faculty (even though it was conceived as a part of the rational soul) of the logically abominable type adopted by John Philoponus (Yaḥyā al-Naḥwī; *ca.* 490 - *ca.* 570).[40]

Avicenna, as I mentioned above, is aware of his failure in the *Shifā'* to cope fully with the individualizing of a 'resurrected' intellect. But the treatment in *Al-Risala al-Aḍḥawiyya* goes little further; the discussion may be 'esoteric', but it is scarcely more 'demonstrative' than that in the *Shifā'*.[41] In the *Aḍḥawiyya* the carrier of human individuality is considered to be the rational soul, which also is the element of a person that is saved at his death. How, though, is it to retain its intellective capacity, to become, indeed, an eternal intellect-in-act? If the rational soul or any essential 'part' of it is really

39. Rahman's well-known views on Ibn Sīnā's idea of the 'ego', expressed in *Avicenna's Psychology* (cited in note 10 above), pp. 12-19, 102-104, and 109-114, were necessarily tentative, for he was speaking there only in relation to Ibn Sīnā's remarks in ch. 15 of the psychological part of the *Najāt* (Eng. tr., pp. 64-68 in the same work, pp. 189-192 in the Arabic text, *ed. cit.* above in note 10). In this place Avicenna merely indicates that the ultimate substrate of experience is in one sense the soul as a whole. The more developed doctrine of the 'ego' is found in the *Shifā'*, *Kitab al Nafs* V: 7, ed. Rahman (cited in note 8 above), pp. 256-257, see especially p. 256, ll. 9-11, a passage that follows upon the analysis of the 'floating man' (see preceding note): 'The referent *(maqṣūd*: 'object referred to') in the knowledge I have about myself, that I am the "I" whom I mean in my saying that I have sensed [something] and that I have intelligized [something] and that I have performed [some] act and that I combine these characteristics [within myself], is another thing, which is what I call the "I" *(anā)*.'

Cf. also note 34 above, see especially the *Shifā'*, *al-Ilāhiyyāt* III: 8 and VIII: 6 (for background), *al-Risāla al-Aḍḥawiyya*, ch. 4, and Pines, 'La conception de la conscience de soi ...', among the references there.

40. Rahman, *Avicenna's Psychology* (cited in note 10 above), pp. 111-114, presents an English translation of Philoponus's remarks on the 'attentive faculty', *cf.* Iohannes Philoponus, *In Aristotelis De Anima libros commentaria* [*Commentaria in Aristotelem Graeca*, XV], ed. M. Hayduck (Berlin, 1897), p. 464, ll. 13 *et seq.* (on *De An.* III: 2, 425b12ff).

41. See above, p. 74 and note 34. The seeming hesitancy of Ibn Sīnā over his doctrines of individuation of immaterial things appears in the *Shifā'* in *Kitāb al-Nafs* V: 3 and *al-Ilāhiyyāt* IX: 7 and X: I, in *al-Ilāhiyyāt* X: 1 (and to some extent in IX: 7) his dubiety is due at least in part to a desire to keep the discussion exoteric (with, of course, hints to the wise), but in the *Kitāb al-Nafs* the doubt seems wholly unfeigned.

reduces to ontology. The problem of knowledge comes to be examined mainly through discussion of the ontological status of intelligibles and intellects. And to this same topic the study of the higher functions of the soul leads also at the end.

The ideas of the 'essential definition' *(hadd)* *(cf.* note 13), the species, and the genus are treated the most interestingly not in the logical books of the *Shifā'* but in the *Ilāhiyyāt*: for it is their mode of being that is principally at issue, and this is an ontological matter. Avicenna's rejection of Platonic Ideas on ontological grounds has already been noted; the ontological content of the problem of intelligible memory, too, will have been evident. Finally, it is in ontology where the problems of psychological theory as such, having been averted earlier, now reappear to do battle.

How can a rational soul be said really to become an eternal intellect-in-act? Would this not require a change from one hypostasis into another, a change of a single subject from one level of being to an entirely distinct one? And would not such a change be entirely inexplicable in anything like Avicennian concepts? It is true that Ibn Sīnā speaks of a child as entering a different species upon gaining its capacity for intellection, but this is simply a case, although a peculiar one, in which a properly prepared matter receives an entelechy that is common to all members of its (new) species.[37] (Scholars who seek to make Avicenna Plotinian must be especially careful on these matters: he never speaks of an individual 'undescended' intellect, and the realization of a rational soul as an eternal intellect-in-act, however grotesquely it may distort Peripatetic views, is simply inconceivable in the cosmology of the *Enneads)*.

The rational soul *qua* individual is not an intellect, whereas *qua* intellect-in-act it cannot be individual. Is the rational soul of a person, however, supposed to be identical with his intellect, and is this in turn to be identical with the intelligibles it receives? If so, there are grave problems here, surely insuperable ones. But in fact, as has been remarked above, Ibn Sīnā often speaks as though the potential intellect is merely a capacity for intellection inhering in something non-passive, namely the rational soul, which in one aspect is the *hegemonikon*, the 'controlling faculty', of the individual. One should note also the self-awareness that Ibn Sīnā attributes even to the so-called 'floating man' *(i.e.*, someone conceived as deprived of all sensory information whatever).[38] Thus he has hinted in several places at an immaterial principle of

37. *Shifā'*, Cairo ed., *al-Ḥayawān* XVI. 1, p. 403, ll. 7-8. 'If a child *(ṣabiy*, lit., 'boy') [duly] endowed with sensation *(ḥassās an)*, becomes [fully] human *(insān an)*, through reason *(nuṭq* = Gk. *logos)*, he progresses by this perfection *(istikmāl*, 'entelechy') from one species *(naw')* to another'. (Because, according to Peripatetic philosophy 'man' is the 'rational *(nuṭqī)* animal', differentiated from all other species by his reason). The present passage is part of a longer one discussed above (p. 52) in connection with the ensouling of the embryo.

38. *Kitāb al-Nafs*, I: 1, *ed. cit.* in note 8 above, p. 16, and V. 7, *ibid.*, pp. 255-256; also, differently expressed, in the *Ishārāt* (*Le Livre des théorèmes et des avertissements*, ed. J. Forget (Leiden, 1892), p. 119).

Whatever the reasons and motivations of Avicenna, and whatever the nature of his mystical tendencies, his theory of the acquisition of knowledge through 'conjunction' with the Active Intellect thoroughly undermines natural philosophy and the sciences, for his solid and integrated account effectively obviates empirical investigation. The examination of *tajriba* in the *Shifā'* may well be intended to save the disciplines traditionally based on experience – Avicenna's own intellectual biography very strongly suggests as much; but the epistemology developed by Ibn Sīnā moves so far in the opposite direction as to become a form of illuminationism. One cannot avoid the impression that the hardships of *tajriba* are really for the intellectually unlucky. Individual immortality has been saved, but empirical research has been made superfluous, at least in essence. Direct intellectual insight can be effective anywhere that *tajriba* can be but is also able to go further and deeper, Avicenna will rightly be understood as saying. Logic and mathematics supply good mental training; noetics, epistemology, and ontology are important for their actual content. The rest of the Greek theoretical disciplines, especially natural philosophy and the mixed sciences, have a lesser value and are perhaps trifling to the best minds. Such is the eventual significance of Ibn Sīnā's treatment of *emperia* for the Greek way of knowledge in the Islamic world. There are strong intellectual and social forces against which the *falasifa* are obliged to make themselves felt, but Avicenna's philosophy turns too much towards illuminationism, and keeps too little of Peripateticism to provide a healthy environment for science. This is the cost that the successors of Ibn Sīnā in *falsafa* and the natural and mathematical sciences will have to meet in order to pay for his success in constructing a system of logic, biology, and metaphysics that gains its coherence through these distinctive theories in psychology.

There is also a price that is exacted within Ibn Sīnā's own philosophy for this triumph. The various problems of his noetic, both psychological and epistemological, have largely been solved; but the solutions that have been reached create further difficulties. The new complications cluster together in the area of ontology. They are taken up in the *Shifā'*, then, in the *Ilāhiyyāt* (meaning, '[science of] things divine', but of course equivalent in meaning to 'metaphysics'), not in the *Kitāb al-Nafs*, which is a physical investigation of the soul, nor in the *Burhān*, the mainly epistemological work placed, however, in the logic *jumla*.

Ibn Sīnā's psychology in general has a tendency to merge into metaphysics. 'How do we think?' and 'how do we know?' are primary questions in his psychological enquiries, and they clearly presuppose the basic epistemological inquiry into the nature of knowledge. But the connection to metaphysics is much more intimate than that, for Avicenna's epistemology in turn largely

d'Avicenne [Mémorial Avicenne, III] (Cairo, 1952) or W. E. Gohlman, ed., *The Life of Ibn Sinā* (Albany, N.Y., 1974).

I have not meant to imply that Ibn Sīnā has no other motives in adopting his intellective theory of *ʿilm* than to save individual salvation, although I have maintained that this is much the weightiest one. But there are indeed further advantages to his account of the external active principle of human intellection. For one thing, all the benefits of a pure, Platonic epistemology are preserved without having the Ideas themselves self-subsistent, which was surely something ontologically objectionable (see *Shifā', Ilāhiyyāt*, VII: 3, and the preliminaries in III: 8; al-Fārābī has already made these points); the Ideas become the conscious contents of the more credibly self-subsistent celestial intellects (which were posited even by Aristotle; see especially *Metaphysics* XII: 8). Furthermore, intelligibles are necessarily immaterial and cannot be retained in a corporeal medium. (What, Avicenna asks, would half a spatially extended abstract man be?) The Active Intellect, however, provides a suitable storehouse from which they can be borrowed conveniently; otherwise, intelligibles would actually have to be abstracted anew each time from remembered images or *intentiones*. The solution of the problem of intellectual memory must be one of Ibn Sīnā's chief grounds of a purely philosophical sort for making the active principle of abstract human knowledge something external.

Finally, the intellection theory of Avicenna allows a quasi-mysticism to be present in his philosophy, and he lives in a period when mystical thought is beginning to pervade Islamic cultural life. Talk of separated intellects and abstract contemplation will help to attract followers and will make the introduction of neophytes to his thought easier to accomplish. It is also very likely that Ibn Sīnā himself finds this aspect of his philosophy satisfying. Certainly he believes it, for he says that he prays (intellectually) for middle terms. It is probable even that he views what I have just called his 'quasi-mysticism' as the only legitimate mysticism. In any event, it comes to be regarded by others as an altogether essential feature of his system.[36]

Syed Hasan Barani in 'Ibn Sina and Alberuni. A Study in Similarities and Contrasts', *Avicenna Commemoration Volume* [*A.H.* 370-*A.H.* 1370] (Calcutta, Iran Society, 1956)). I find the tone authentic.

The poem is also quoted by Seyyed Hossein Nasr in *An Introduction to Islamic Cosmological Doctrines* (Cambridge, Mass., 1964), p. 183, and again in his *Three Muslim Sages* (Cambridge, Mass., 1964), p. 41, each time in a discussion of Ibn Sīnā and Islam; either will provide an interesting preliminary account.

36. In a positive sense, by the Iranian philosophers beginning with Naṣīr al-Dīn al-Ṭūsī and his neo-Avicennianism in the mid-thirteenth century, and culminating with Mullā Ṣadrā (Ṣadr al-Dīn al-Shīrāzī, ca. 1573-1640) and his synthesis of Ibn Sīnā's philosophy and the theoretically developed sufism of Muḥyī al-Dīn Ibn ʿArabī (1165-1240). On the modern controversy over Avicenna's mysticism see below.

Ibn Sīnā mentions praying for middle terms in his autobiography, and his ideas on the nature of prayer are expressed compendiously in the essay 'On Prayer'; both are conveniently accessible in English in A. J. Arberry, tr., *Avicenna on Theology* (London, 1951). There is no critical Arabic text for the *Risālat al-Ṣalāt*, but for the autobiography see A. F. al-Ahwānī, ed., *Aperçu sur la biographie*

Avicenna's doctrine of individual salvation, although far removed from Qur'ānic teachings, is in the end a conviction that springs from religious rather than philosophical motives. Responsibility for his ideas lies here with his thoroughly (if not always stringently) Muslim surroundings, not with his reading of the philosophers.

The examples from embryology and epistemology considered in this paper attest the fundamental importance of personal immortality to Avicenna's philosophy. They should help to confirm my introductory remarks concerning the dialogue in medieval Islam between the more traditional groups of religious intellectuals and the Muslim philosophers. I trust they will also begin to show why it may be asserted that these philosophers, even Ibn Sīnā, who is more difficult to analyse than some, consider their thought not merely to be acceptably Muslim but to be the one true interpretation of their religion.[35]

4 above). The *Najāt* offers nothing of real interest, except where it repeats the *Shifā'*, but a bit may be gleaned from Rahman, tr., *op. cit.* in note 10 above, chs. 11 and 12, *passim* (= pp. 182-184 of the Arabic text, *ed. cit.* in the same note).

Certain modern studies may also be consulted: Louis Gardet, *La pensée religieuse d'Avicenne (Ibn Sīnā)* (Paris, 1951), pp. 88-94 and 98-105 in ch. 3, pp. 129-131 in ch. 4, and 145-183, *passim*, in ch. 5; *idem*, *La connaissance mystique chez Ibn Sīnā et ses presupposés philosophiques* [= *Memorial Avicenne*, II] (Cairo, 1952), which is a preliminary version of ch. 5 of the preceding, but with certain passages in Arabic included in the notes — pp. 7-49; Shlomo Pines, 'La Conception de la conscience de soi chez Avicenne et chez Abu'l-Barakāt al-Baghdādī', *Archives d'histoire doctrinale et littéraire du moyen-âge*, 20-21 (1953-54), pp. 20-99, one of the few really excellent studies on any aspect of Ibn Sīnā's thought; and Francesca Lucchetta, 'Introduzione' to the *Adḥawiyya*, *ed. cit.* in note 4 above.

Cf. also note 39 below. It should be pointed out that the *Adḥawiyya* presents doctrines that are consistent with what the sophisticated reader of the *Shifā'* would expect: bodily resurrection is dropped, individual salvation is kept, and *ittiḥād* is still rejected — there is only intellectual contemplation-in-act of the One as duly 'reflected'.

The aspects of *ma'ād* that relate to moral purification are not taken up in this paper, nor is the question of the original individualization of the rational soul for its body considered (but it should be noted that this is done on the basis of the material attributes of the embryo), although both topics are important and are treated in both the *Shifā'* and the *Adḥawiyya*.

As regards the traditional Peripatetic doctrines in this area of ontology, it must be said that Aristotle nowhere provided an adequate examination of the 'governing' part (or aspect) of the soul, or gave a focused analysis of the relationship of the rational soul to the intellector of the intellect to the intelligibles. For the Aristotelian view that a *nous* as such is identical to its *noeta*, see, *passim*, *De An.* III: 4, 5, and 7, and *Meta.* XII: 7 and 9.

35. Virtually all the *falāsifa* (a notable exception being Muḥammad ibn Zakariyyā al-Rāzī, Lat. Rhazes, ca. 854 - ca. 930) feel their philosophy and their religion to express the same Truth; a more precise statement than this, however, would require lengthy elaboration. Many of the consequences of that belief are expressed in their political philosophies, on which see, first, the Islamic part of Ralph Lerner and Muhsin Mahdi, eds., *Medieval Political Philosophy: A Sourcebook* (New York, 1963). The rôle of Islam in the life and thought of Ibn Sīnā is peculiarly hard to assess, not least because of his ability to be 'all things to all people'. Louis Gardet in *La pensée religieuse d'Avicenne* (cited in the preceding note) has devoted a book essentially to this subject; for his conclusions see esp. pp. 201-206.

There is a Persian poem attributed to Ibn Sīnā that ends, 'I am the unique person in the whole world and if I am a heretic/Then there is not a single Muslim anywhere in the world' (Englished by

which enters the embryo, and the 'acquired intellect') are necessary in the philosophy of Ibn Sīnā in order to explain personal intellectual immortality. Many of the contortions in Avicenna's psychology, his metaphysics, and even his biology are in fact introduced to this same end.

Ibn Sīnā claims that the 'saved' human intellect remains individual in its eternal state of 'conjunction'. The only possible way for him to justify this assertion philosophically is to elaborate his conceptions of the rational soul and of the passive intellect. The two seem to me to be effectively identical, but let me for the moment call the entity which achieves 'resurrection' and immortality the 'rational soul' and let the passive intellect be simply the capacity for intellection which is attached to it. The rational soul, first of all, is very 'active' in certain respects, even if its most important function is to be conscious of the intelligibles of which it is receptive *qua* intellect; it serves the same purpose, indeed, as the *hēgemonikon* of Aristotle (and in this rôle is less ambivalently described than was Aristotle's 'governing power'). The Avicennian notion of the 'ego' is closely connected with the idea of the individual rational soul (which in essence has the logically difficult attribute of being an individuated *intellect)*. Unlike Aristotle's *nous*, the rational soul of Avicenna has the faculty of receiving or sharing, but not simply of becoming, the intelligibles; the human *ʿaql*, when Avicenna means by it, as he very often does, either the rational soul or at least something more than a pure intellective faculty, never is identical to its *maʿqūlāt*. The preserving of the identity of the 'resurrected' rational soul *cum* intellect is a major requirement of his authentic teachings on salvation and not (in contrast with his remarks in the *Ilāhiyyāt* of the *Shifā'* on the miraculous resurrection of the body) a view put forward for the sake of religious expediency. But, in my opinion, Ibn Sīnā is able to make only a start on the necessary analysis. He does seem more confident about his doctrines in *Al-Risāla al-Aḍḥawiyya fi'l-Maʿād* than in the *Shifā'*, and in the *Aḍḥawiyya* he speaks primarily in terms of the (rational) soul rather than the intellect. Now it is certainly true that the Active Intellect as *dator formarum* is also the source of the rational soul as the form of the individual human being, and thereby as the form of those other, intelligible forms that he will receive – as Aristotle said, the mind is a 'form of forms' *(De Anima* III. 8, 432a2). The obvious ontological difficulties are not solved in a demonstrative way, however, in any of Avicenna's writings that I know.[34]

34. On the problems in ontology, and especially the individuation of intellects, the following are among the principal discussions, in the *Shifā'*, *al-Ilāhiyyāt* III: 8 (on the intellect as substrate for 'quiddities', *māhiyyāt*), VIII: 6 and IX: 5 (background), IX: 7, and X: 1 (relevant but disappointing), and in the *Kitāb al-Nafs*, V: 3 (particularly), and also V: 7, *passim* – but note Avicenna's warning (p. 238, *ed. cit.* in note 8 above) that the condition of the soul after death does not belong to the subject-matter of natural science (but rather to metaphysics), and in *Al Risāla al-Aḍḥawiyya* (an esoteric but scarcely apodeictic work), chs. 1, 4-6, and parts of ch. 7 (pp. 190-209 and 214-223, *ed. cit.* in note

ported by a general view of Aristotle's ontology and epistemology based upon passages found in *De Anima* III: 5, *Metaphysics* II: I and XII: 7 and 9, *Nicomachean Ethics* X: 7, and elsewhere (not excluding the 'Theology of Aristotle'), as well as in the works of Aristotelian commentators such as Alexander of Aphrodisias and Plotinus (for so he was regarded). Ibn Sīnā believes that this tradition of thinking, supplemented by various Islamic insights, has the Truth. But for individual tenets within that structure he feels no real need (I am persuaded) for particular textual justifications. Indeed in pressing his own views Avicenna usually finds the specific texts of others simply convenient props or annoying barriers.

Apart from their deviation from the purer Aristotelianism, however, what has been learned of general significance about the doctrines in Book III, chapters 5 and 6, and Book IV, chapter 10, of the *Burhan*? *Tajriba*, one has been told, develops through the products of estimation as they are retained with increasing orderliness in the memorative faculty. This 'experience' is 'illuminated' by the Active Intellect in such a way that the corresponding intelligibles are made present to the human potential intellect - which thereupon becomes an intellect-in-act, the *ʿaql mustafād* or 'acquired intellect'.

The careful noetic built up in the *Kitāb al-Nafs* is consistent with the last of the accounts in the *Burhān*, which indeed smoothes the way for it. The essence of Avicenna's explanation when it has finally been consolidated is simple: through the workings of sensation and imagination and the formation, ultimately, of 'experience', the grasping of true, intelligible knowledge 'from without' is occasioned; but this knowledge can be conserved only in the separate Active Intellect, and whenever an individual person shares in these intelligibles his intellectual faculty must be conjoined to the higher intelligence. The absolutely intellectual and incorporeal nature of human knowledge has thus been upheld, while a rôle in acquiring knowledge has nevertheless been found for man's sensory faculties.

The main consequence of keeping true cognition independent of things bodily, as Ibn Sīnā intends it, is the possibility of immortality for the individual intellect. It is his belief, already examined briefly above, that a person's soul eventually can reach a point where it no longer depends at all upon corporeal faculties in attaining the intelligibles, but is in fact prevented by the body from prolonging its periods of intellectual contemplation. This independence is to be achieved by constantly actualizing the rational faculty as an intellect – through 'conjunction', and in most cases, at least at first, from a basis of 'experience'. To a soul thus elevated the death of the body is to come as a release that will allow it to enter the supremely happy condition of eternal intellection.

The rejection of purely empirical theories of knowledge and the postulating within each human being of two entities from above (the rational soul itself,

(6) involve direct intellectual *taṣdīq*. *Tajriba* enters explicitly into (5), but also, implicitly by way of *taṣawwur*, into (2) and (3). It scarcely need be added that in every case the unexpressed phrase 'from the Active Intellect' is to be understood after the verb 'receive' or its equivalent.

Much has thus been said about 'experience' by Ibn Sīnā in those chapters of the *Burhan*. But however helpful *tajriba* may be, in the end it does nothing that is absolutely essential. This conclusion is already implied clearly enough, except in one case; but it holds, as one learns elsewhere, even for *taṣdīq* in respect of propositions like 'scammony purges yellow bile'. *Tajriba* cannot serve as a proper originative source for *ʿilm*. Here in Avicenna's system with regard to the acquisition of knowledge through experience, even more than earlier on with regard to the ensoulment of the human embryo, there is a lesson to be drawn concerning Ibn Sīnā's attitude to Aristotle. The greater part of the *Shifā'*, as was said above, follows the standard arrangement of the Aristotelian *corpus*. Yet within this minutely structured framework of topics, Avicenna is his own man: it is the questions and not their treatment that are routinely taken over. Ibn Sīnā philosophizes in a well-defined tradition, but departs from his predecessors, from Aristotle himself, not merely in details but in major doctrines. In the accounts of *tajriba* that have just been examined, the First Teacher's opinions are first twisted, then ignored. The radical dichotomy between the sensible and intelligible worlds is stoutly maintained. Regardless of his esteem for Aristotle, Avicenna refuses to allow the senses or anything that is at all corporeal to create genuine, intellectual comprehension. Despite the soothing words of the preliminary discussion in III: 5, *empeiria/tajriba* is allowed only to lead towards, not actually to produce authentic knowledge. Notions from 'experience' cannot have any actual connection with abstractions proper. In some instances 'experience' may become a necessary cause of the acquisition of intelligibles; but it is never, as it was for Aristotle in the *Posterior Analytics*, the stuff out of which true knowledge is refined, the actual origin of the arts and sciences, which is continuous with them. The real source of *ʿilm* as conceived by Ibn Sīnā is something entirely different, the intelligibles subsistent in act in an eternal higher intellect. Although not at all an unprecedented rewording of Aristotle, in the context of the [*Kitāb*] *al-Burhān* this is a boldly consistent one. The empirical theory of knowledge is effectively destroyed in a chapter that pretends to save it! The rational soul, which comes to the embryo 'from without', does indeed require that second entity 'from without' to make it think; only with the 'acquired intellect' is it really rational.

The un-Aristotelian treatment of the Aristotelian topics is itself very coherent, as the reader of Avicenna gradually discovers. Not that Ibn Sīnā would regard his own philosophy as anti-Peripatetic: quite the contrary. The liberties taken with *Posterior Analytics* II: 19 and *Metaphysics* I: 1 may be sup-

in IV: 10 one does not possess an integral, esoteric presentation of the theory of how the human mind obtains knowledge (although Ibn Sīnā goes well beyond the professed goal of the chapter, which is only to describe the acquisition of primary premisses). What one does have is an accurate delineation of the main tenets.

Reflection on the whole of Ibn Sīnā's handling of the acquisition of knowledge in the *Burhān* leaves the impression that all is not well, even when allowance has been made for the peculiarities of the method of presentation. Inconsistencies remain between the discussions in III: 5 and IV: 10. There is no hint in the earlier account that *tajriba* may be considered a cognitive state, nor is this a matter which can be corrected by a simple elaboration. Again, there is no indication in III. 5 that 'experience' has a rôle to play in *taṣawwur*, despite the not inconsiderable discussion there of *taṣawwur* and the senses. The earlier conceptions of *istiqrā'* and of the *tajriba* that generates 'assent' *(taṣdīq)* to premisses about the physical world *(e.g.*, that 'the lodestone attracts iron') Ibn Sīnā does not revise, and the necessary modifications are left implicit. Nor, as was said, does he carry out a frank examination of the necessity of the sensory and 'estimative' preparations for intellection that he has described.

There is a further, more general shortcoming. Avicenna's analysis really amounts to little more than a mere exhausting of logical possibilities, for he pays scant attention to conditions which actually may determine the occurrence of the processes that he has identified. (This of course is also an obvious flaw in Aristotle.) Especially to be noted is the case of *taṣdīq* with regard to composite universals, where it is unclear which of the two possible routes is to be followed in any particular instance – whether sensory (including 'estimative') combination of 'images' is to give rise to *taṣawwur* of the compound intelligible, which is then subject to *taṣdīq*; or whether sensory processes are to lead to *taṣawwur* of incomposite intelligibles, which are afterwards combined intellectually into the compound intelligible.

From the material that Avicenna does present, however, one is able to extract a list of six intellectual processes which he believes operate to acquire *'ilm*. The intellect by its nature may, he says: 1) receive unmattered, incomposite intelligibles; 2) pare the 'images' of immattered forms and grasp the corresponding incomposite intelligibles; 3) receive primary premisses by way of abstraction from compounded 'images'; 4) acquire primary premisses through the combination of two intelligibles which it knows directly by innate disposition *(fitra)*; 5) gain secondary premisses through *tajriba* and the recognition of certain conjunctions as essential rather than accidental; and 6) obtain derivative premisses (in what Aristotle designated '*epistémé*', in the narrowest sense) by syllogistic combination of intelligibles. Processes (1) and (2) relate solely to *taṣawwur*, the rest to both *taṣawwur* and *taṣdīq*; (4) and

khayāl to prevent his statements from seriously misleading the reader. The explanation was not complete, but neither was it actually wrong, he would claim; moreover, he would certainly say that it was the proper and most appropriate way to present the material at that stage in the exposition. After all, to mention only the most difficult point, the *intentiones* are still sensory and bodily as compared with the radically different intelligibles.

The treatment of *tajriba* and *ʿilm* in the *Kitāb al-Burhān* is not a wayward example; on the contrary, it actually represents Ibn Sīnā's regular manner of handling a difficult subject. No more theoretical armament than necessary is brought to bear in a given situation. Hence it is clear that to glean a theory from Ibn Sīnā's explanations where it is not the main subject at hand is a very dangerous course indeed, and to find contradictions between such subsidiary accounts is simply illegitimate.

But how can the reader know that in IV: 10 he has come to an essentially complete portrayal of the rôle of experience in the attainment of knowledge? A preliminary answer is that when compared with the presentations in III: 5 and III. 8, at least, this one immediately can be judged preferable simply because it is fuller and fits better with the rest of Avicenna's philosophy. The decisive condition which is met here, however, is that the last account finally reproduces the entire psychological scheme as it appears in the main analysis of the workings of the soul, by which I mean the description of the intellect and its subordinate faculties found in the *Kitāb al-Nafs* in the physics *jumla* of the *Shifā'*. (Conversely, from his knowledge of the *Burhān* the reader can see immediately that the summary of the functions of *tajriba* in the *Kitāb al-Nafs*, V: 3, reproduced in chapter 11 of the psychological part of the *Najāt*, provides nothing more than a glimpse of the subject in a special context and should be accorded virtually no weight (see note 10 above).)

The more delicate question arises whether even in the principal discussion of psychological theory certain esoteric doctrines are being suppressed. But there must be a discernible motive on the part of Ibn Sīnā before the historian may allow himself to entertain that suspicion: for example, that the intended readers of the treatise are insufficiently advanced or religiously too unenlightened to understand Ibn Sīnā's real views. In this case no such considerations seem to apply. Therefore, since the treatment of empirical knowledge in *Burhān* IV: 10 is fully compatible with the system expounded in the rest of the *Shifā'* and, moreover, in the *Ishārāt* and elsewhere it should indeed portray his doctrines in a reliable way. This is not to say that one finds here a straightforward, closed, or exhaustive explanation. The actual positions of Avicenna have to be teased out of the text, which superficially aims to 'save' Aristotle's opinions. No overt alterations are made to the assertions in III: 5, although more than one is implied. The embarrassing but essential question of the necessity of sensory information and of 'experience' is not explored. So even

estimative and retentive faculties is a new, intermediate level of cognitive object, the *ma'ānī (intentiones)*. More abstract and analytically powerful than the sensory images even of the cogitative faculty, they are nonetheless corporeal and only quasi-universal; so the *ma'ānī* count ultimately as 'sensible', not 'intelligible'. 'Experience' *(tajriba)* results from the accumulating and sorting of the *ma'ānī* by the soul. It now transpires, moreover, that mere sensible forms normally need to be refined into *intentiones* for intellection to occur. Only then are the intelligible species and their relationships clearly enough 'reflected' (if a neo-Platonic term used in the *Adhawiyya* may be borrowed) that individual human intellects may be stimulated to the grasping of the actual intelligibles. This again accords with the *Kitāb al-Nafs (q.v.*, Bk. IV, ch. 3).

Let that suffice for 'experience' as it is explained in *Burhān*: IV:10. A comparison with certain features of what was said on the same subject in Book III will provide a striking illustration of a particularly important characteristic of Avicenna's expository methods. It must be stressed first that Ibn Sīnā does not intend to describe a different doctrine of the acquisition of knowledge *via* experience in Book IV of the *Kitāb al-Burhān* from what he has done earlier on; he has not changed his theory, nor would he admit to being gravely inconsistent in his presentations – despite the fact that it would be difficult to infer a rôle for combinative imagination from the earlier accounts and impossible to do so for 'estimation'. It is the case, rather, that Avicenna customarily deploys only as much of his full theory as is absolutely requisite for the immediate objective.

His practice in this respect is partly a matter of instructional method and to some degree of mere convenience; it is also a natural correlative of his policy of gradual disclosure (in religiously sensitive or highly abstruse topics) of a fully 'esoteric' doctrine to an increasingly restricted audience of the philosophically élite. Consequently, the works of Avicenna are fraught with difficulties for any one who wishes to learn about his views on some specific subject without studying his system as a whole. For the intellectual historian the most relevant implication is the obvious one, that an understanding of one of Avicenna's doctrines must always be grounded upon the principal discussion of that teaching (if a full treatment exists) and never upon inferences drawn from a series of passing mentions. (This restriction supplements two others: that one must ignore, for the most part, rhetorical presentations whenever a dialectical or demonstrative one exists, and that one must 'read between the lines' in order to recognize places where esoteric doctrines may be lurking – the latter by no means a particularly difficult feat for a reasonably experienced and unprejudiced student.)

In the case of the empirical acquisition of knowledge, Ibn Sīnā in his earlier descriptions has depended upon the latitude of meaning in the terms *ḥiss* and

without intellectual help. (But, Ibn Sīnā reminds his readers, what the *wahm* discerns is one thing, what the intellect grasps is another.) This discrimination is accomplished, Avicenna says, not by sense-perception proper but specifically by estimation. One may infer that the recognition of natural species is in fact an elemental function of 'experience'.[33]

Like Aristotle, Ibn Sīnā brings his analysis to a halt when he has identified the faculty which acquires abstract and indemonstrable knowledge; any further investigation of the means of knowing belongs elsewhere, that is to say in the study of psychology. In the *Posterior Analytics*, sensation and its further development *via* memory and experience seem to have formed a necessary and sufficient source for all intelligible knowledge; but to Ibn Sīnā a sensory foundation is necessary only in certain areas of enquiry (and for some few people not even there), and in no case can it become a sufficient principle for intellection. The incomplete human intellect, in Avicenna's view, always needs external help to possess actual intelligibles. Moreover, the sensory and 'estimative' aids become obstacles to any intellect that has already developed its capacities and come to know its way about in the intelligible world. Things relating to sense are to be discarded as quickly as possible, Ibn Sīnā maintains; dependence on corporeal faculties can lead one's soul only to torment in an afterlife where bliss is intellectual.

Tajriba is the final result of sifting and arranging the *intentiones*, but upon the *intentiones* the light of the Active Intellect must shine if the mind is to acquire real knowledge. Although still subject to all the detailed qualifications presented before, Avicenna's final doctrine can be summarized quite simply: when something the intellect is supposed to know is displayed before it in suitable 'images', it does know it, in an intelligible way – for that is its peculiar power as an intellect. Of such 'images' the most highly developed and directly stimulating ones are the sorted *maʿānī*, the ordered *intentiones* that are held in the retentive faculty and constitute 'experience'. Prepared by 'experience', the soul has become ready for its intellectual faculty to be actualized from without, ready to grasp the intelligibles *in actu* through conjunction of its individual potential intellect with the eternally actual Active Intellect.

This discussion in Burhān IV: 10 provides the last instalment of Ibn Sīnā's explanation of 'experience'. Here he correlates the analyses of the earlier chapters with the psychological theories of the *Kitāb al-Nafs*. The previous treatments are elaborated in such a way as to disclose the parts played in the sensory half of human cognition by two additional 'active' faculties, the combinative imagination *(al-mufakkira*, the 'cogitative' faculty) and the estimative faculty *(wahm)*, and by the repository for the products of the *wahm*, the retentive or memorative faculty *(al-ḥāfiẓa, al-dhākira)*. Associated with the

33. *Burhān*, Cairo ed., IV.10, p.332, ll. 5-15

Under the influence of Aristotle's exposition in *Posterior Analytics* II: 19 (esp. 100a3-9), *tajriba* has become in *Burhān* IV: 10 not merely the process similar to 'a mixture of sensory induction *(istiqrā')* with intellectual deduction' that was described in III: 5, but also a cognitive state of the soul established by the well-marshalled contents of the retentive faculty. *Tajriba* has been made the nearest possible Avicennian equivalent of Aristotle's *empeiria*, which 'develops out of frequently repeated memories of the same thing' (100a4-6) and from which originate the arts and sciences (the latter contention being explained more fully in *Metaphysics* I: 1, 980b25-981a12).

An analogous change should almost certainly be made retrospectively in the interpretation of Avicenna's notion of *istiqrā'*: although sensory in a general way it too must belong primarily to 'estimation'. Indeed in the light of statements elsewhere in the *Shifā'*, especially regarding mathematical examples, this is a safe inference and not simply a conjecture.[32]

Through the discussion in *Burhān* IV: 10 the word '*tajriba*' has come to denote the resultant state of the soul as well as the process, or family of processes, from which that state arises. Moreover, *tajriba* now may be described in another way, as the settled judgements in the retentive faculty that have been obtained through 'estimation', and thus ultimately from a sensory basis

Having presented his alternative to Aristotle's explanation of how universals, especially the primary premisses, are acquired, Ibn Sīnā turns for the first time in the chapter to an explicit consideration of Aristotle's text, to the analogy drawn by the 'First Teacher' between the coming-to-a-stand of a universal in the soul and the coming-to-a-stand in their proper battle-formation by troops after a rout *(Posterior Analytics* II: 19, 100a12-13). Avicenna concedes all that he can, but it is not really very much Knowledge *('ilm)* and the intelligible universal form are delineated little by little from sensible singulars, he agrees, and when these have been joined together, the soul acquires upon this basis the universal as such and then discards the sensory antecedents. Although the universa lman is somehow contained in the individual man reported by the senses, the notion 'man' *qua* sensible is 'diluted', Avicenna says; or, he continues, using a different and favoured metaphor, the sensible 'man' must be 'pared' by the intellect (so as to remove the 'husks' and permit access to the intelligible kernel). Working upward from the sensibles, however, the *wahm*, both in higher animals and in man, is able to distinguish between individuals of one biological species and those of others

32. See the references given on pp 82-84 in Shlomo Pines, 'Philosophy, Mathematics, and the Concepts of Space in the Middle Ages', *The Interaction between Science and Philosophy*, Y. Elkana, ed. (Atlantic Highlands, New Jersey, 1974), pp. 75-90. The relationship of 'mathematicals', mathematical reasoning, and the *wahm* in Ibn Sīnā's system is more complicated than it appears there, however I hope to publish an article on this topic with full documentation, especially from the *Shifā'*, in the reasonably near future.

demonstrables.[30] (The function of *tajriba* in *taṣawwur*, it should be noticed, emerges here for the first time).

The analysis is rounded off by Ibn Sīnā's statement that the other composite universals, *i.e.*, those that are not first principles, gain assent *(taṣdīq)* from the intellect either by means of *tajriba* or by syllogistic demonstration through a middle term.[31] *Tajriba* in this case must relate to the extraction as *intentiones* of that which is essential in the sensorily apprehended conjunctions among things and from which the intelligible relations can be fully abstracted. In looking back it seems that this is the process that was meant in III: 5, and that scammony's purging of yellow bile and the other examples there were instances of this particular utilization of *tajriba*. It is made where there can be no middle term, yet where the composition of the simple intelligibles does not in itself necessitate assent. Finally, one may infer that the apprehension of middle terms also can involve *tajriba* in the way just introduced, or that, instead, it can be purely intellectual.

The account in *Burhān* IV: 10 is a disjointed one, even more dispersed in the original than here. But, especially when supplemented, as indeed it must be, by a reading of the *Kitāb al-Nafs* (to which the reader is explicitly referred at the end of the chapter), it is a very substantially coherent treatment. Insofar as *tajriba* is concerned, one has gradually been informed that it assists in *taṣawwur* with respect to intelligibles generally and in *taṣdīq* with regard to primary premisses. *Tajriba* of this kind is generated from a sorting of the contents of the retentive faculty, so that the products of the *wahm* become almost abstract. In creating *taṣdīq* about secondary premisses concerning the observable world, *tajriba* is the most usual means and often it appears a necessary one. Here again it would be pre-eminently the *maᶜānī* that are involved, although Avicenna leaves this as an inference to be made by the reader. 'Experience', in short, is the ultimate cognitive product of the sensory level of the soul and is what the human intellect can use best when seeking the actual intelligibles from the Active Intellect.

30. *Burhān*, Cairo ed., IV.10, p. 331, ll. 16-20. *Cf. Post. An.* II:19, 100a3-9, and also *Meta* I 1, 980b25-981a12. Although in Aristotle's accounts, 'memory' is always *mnēmē*, in the Arabic versions it is sometimes translated by *dhikr*, sometimes by *ḥifẓ*. No Arabic MS of *Meta.* I (i.e., A). 1 is known to survive, but the main source for the Arabic text of the *Posterior Analytics*, the translation by Abū Bishr Mattā ibn Yūnus, is extant. The Arabic translation of 100a3-9 (ed. Badawī, *op. cit.* in note 24 above, vol. II, pp. 463-464) has both *dhikr* and *ḥifẓ*, thanks to the rhetorical style favoured by the Baghdad philosophers; the alternative word, moreover, is given as a variant in each case. Perhaps the best reading is indeed that which is most suited to Avicenna's purposes, *viz.*, that in which *dhikr* is connected with sensation and *ḥifẓ* with 'experience'. The most important phrase is rightly worded, in any case (with no variants given in the one – albeit very authoritative – MS used by Badawī). *al-aḥfāẓu'l-kathīra fi'l-ᶜadad hiya tajriba wāḥida* ('many rememberings produce (lit., 'are') a single "experience"', where 'rememberings' comes from the root [ḥ-f-ẓ]).

31. *Ibid.*, p. 332, ll. 1-3.

The incomposites are subsequently related to each other with the help of the active imagination (*i.e.*, the cogitative faculty). A commentator on the *Shifa'* would like to add here 'and the help of the *wahm*'; but this does not appear in the text, and it is conceivable that compound *intentiones* are to be obtained only by abstraction from sensible forms joined together in the *mufakkira*, instead of through direct combination in the *wahm*. Whichever be the case, Ibn Sīnā states that composites then appear among the *maʿānī*; and when one is produced that the intellect should know without instruction, it does know it, and in a fully abstract and intelligible way. Where necessary, the intellect tries out ([j-r-b], II) the now intelligible premiss, in order, it seems, to comprehend it completely. So, Ibn Sīnā concludes, *taṣdīq* often arises from the senses by way of *tajriba*. The term here may only refer to the 'trying out' that has just been mentioned; and it must designate the same kind of 'experience' as that which was discussed in Book III, for at this point Avicenna actually draws the reader's attention to his earlier treatment of *tajriba*.[28]

Specifically as regards first principles, apprehension *(taṣawwur)* occurs *via* sensation, cogitation, and estimation, Ibn Sīnā now asserts; through these the incomposites are 'imaged' and then combined so as to be apprehensible *qua* composed. After being grasped in this way the composites are intelligized in essence, and assent *(taṣdīq)* takes place spontaneously with respect to correctly related intelligibles – provided that the intellect thus prepared by sensible forms and *intentiones* be conjoined to the 'divine emanation', *i.e.*, to the Active Intellect. These 'first principles' or 'first cognitions', as Avicenna calls them here, are what in the *Kitāb al-Nafs* he terms 'primary intelligibles' and describes as 'the basic premisses to which assent *(taṣdīq)* is given without being obtained ([k-s-b], VIII) [by any process] and without any awareness that assent might be withheld'.[29]

Ibn Sīnā provides further and very enlightening information in this chapter. The retentive faculty, he says, is reinforced by repeated sensory impressions that resemble each other *(maḥsūsāt mutashābiha mutakarrira)* – indirectly reinforced, for first (in a necessary step rather confusingly omitted here) the *wahm* must act upon the sensible forms. In the next stage, 'experience' *(tajriba)* is reinforced – nay effected, Ibn Sīnā adds, strengthening his assertion – by repeated *intentiones* that resemble each other *(maḥfūẓāt mutashābiha mutakarrira)*. The *maḥfūẓāt* are literally the 'contents of the retentive faculty', but these are, of course, the *maʿānī* or *intentiones* that have been retained by the soul. And then from 'experience', Avicenna concludes, the intellect snares universals, either incomposite or combined, as objects of apprehension *(al-mutaṣawwara)* and composite universals as objects of *taṣdīq*, if they are in-

28. *Ibid.*, p. 331, ll. 7-10.

29. *Kitāb al-Nafs* I·5, *ed. cit.* in note 8 above, p. 49; the passage is also contained in the *Najāt* (Arabic text, *ed. cit.* in note 10 above, p. 166, in Rahman's Eng. tr., cited in the same note, p. 34).

in its restricted technical sense. This meaning is explicitly utilized in IV: 10, where *khayāl* designates the lower, 'passive' imagination or 'representative faculty', which, as one is told in the *Kitāb al-Nafs*, serves as the memory for the synthesized sense-reports assembled by the 'common sense' and, when required, 're-presents' these integrated images for use by other faculties. But there is also a higher, 'active', combinative imagination, able to divide, recombine, and manipulate images, and thus 'imagine' in the usual modern sense; Avicenna calls it the 'imaginative faculty', *(al-mutakhayyila)*, or, without ambiguity, the 'cogitative' (*mufakkira*) faculty. The *mufakkira*, like the *khayāl* and, as noted earlier on, the *wahm*, is fully described only in the *Kitāb al-Nafs*. Unlike the estimative faculty, however, the combinative imagination is by no means original with Avicenna. Even as early as Aristotle there was a similar distinction which was made, namely that between 'sensory' and 'deliberative' imagination *(e.g.*, in *De Anima* III: 10-11; *cf.* also the analysis in *De Memoria et Reminiscentia* as a whole).

The introduction of 'active imagination' and 'estimation' in *Burhān* IV: 10 elaborates the analysis of the acquisition of knowledge into a form coherent with the theoretical psychology developed farther on in the *Shifā'* in the *Kitāb al-Nafs*. The *mufakkira* and the *wahm*, while remaining on the sensory side of the cleft between sensation and intellection, do help to narrow it; sensory and intellective processes never can be continuous with each other in the system constructed by Avicenna, but he is reasonably successful here in his attempt to align them with precision in areas where *tajriba* has brought them close together.

The fuller descriptions in IV: 10 emphasize a second sort of *taṣdīq*, barely noticed previously, where the 'acceptance' follows automatically upon the 'apprehension'. It is this kind of acceptance which Ibn Sīnā assigns to first principles. By these he means the indemonstrable universal statements that serve as axioms for thought in general or for individual sciences. The example which he gives here is the idea that the whole is greater than the part; elsewhere he mentions the rule that quantities equal to the same quantity are equal to each other and the laws of contradiction and of the excluded middle.

A full synopsis seems the only satisfactory way to explain the place alloted to *tajriba* in the final scheme. From the contents of sense-perception, Ibn Sīnā says, two kinds of cognizable entities are obtained: the sensible forms, stored in the passive imagination, and the *intentiones (ma'ānī)*, extracted by the estimative faculty and stored in the retentive faculty. These forms and *intentiones* are confirmed, or 'reinforced', in modern terms, by further sense-perception and estimation. From them are apprehended incomposite universals (of entities sensible in essence).[27]

27 *Burhān*, Cairo ed., IV:10, pp. 330, l. 17 - 331, l. 6.

premisses by means of experience *(tajriba)*, he adds. But even in these cases, where sensation indeed allows one to reach the universal premisses, the actual cognizing of them is not by sensory means.

The carefully delayed attack against Aristotle's position comes at last in *al-Burhān*, IV: 10[25] – predictably, for this chapter occupies the place corresponding to *Posterior Analytics* II: 19. Avicenna, clearly, must oppose the wholly empirical theory of knowledge which there received Aristotle's most lucid exposition.[26] No mention of this delicate fact falls on the innocent ears of the reader, however; the offending doctrines of the First Teacher are simply not indicated. Instead of such argumentation, Ibn Sīnā at last provides a full if discontinuous summary of his own theory.

The object of the chapter is indeed the same as that of Aristotle's: the identification of the faculty of the human soul whose business it is to know primary premisses without being taught and the discovery of the manner in which this faculty becomes operative. For both men the entity sought is, in fact, the intellect: the *nous* (as 'intuitive reason') in the case of Aristotle – a faculty immanent and complete in itself, at least in this analysis; and the potential intellect *(ᶜaql bi'l-quwwa)*, which is actualized by the external Active Intellect, in the case of Avicenna. The most interesting divergence here between their doctrines is that which concerns the relationship of knowledge to experience. Before these accounts can be compared, however, Avicenna's needs to be studied with some care, the more so as it departs very considerably from what might be expected on the basis of Book III.

Ibn Sīnā now presents an integrated epistemological and psychological description of the acquisition of basic premisses. In the apprehension and acceptance of these first principles, he explains, other faculties assist the intellect, *viz.*, the external and internal senses. Among the latter this time he names the 'estimative' faculty, whose quasi-universal *intentiones* he discusses, the special memory for the *intentiones*, and two carefully distinguished imaginative faculties.

Whenever Avicenna spoke of imagination in *Burhān* III: 5 he used only the term '*khayāl*' and its derivatives and talked in a way appropriate to *khayāl*

25. *Burhān*, Cairo ed., pp. 330-333.

26. *Post. An.* II.19, 99b20-100b17. This account is complemented by that in *Meta.* I:1, 980a27-981a30.

Aristotle's 'empiricism' is, finally, a matter of interpretation, but the opposed view must take account not merely of these two passages, and the two already discussed by Ibn Sīnā in *Burhān* III.5 and III.8, but a great many others, all of which are ignored here. The idea that Aristotle believed intelligibles to be abstracted from sensory 'imagings' by an internal active principle of human intellection, and to be stored, *in potentia*, in those images, receives powerful support from such texts as *De Anima* III.3, 432a 7-10, III. 7, 431a 14-20 and b2-19, and III; 8, 432a3-14, and *De Mem. et Rem.*, 1, 449b30-450a 14. Avicenna deals with these in connection with other issues, mainly in the *Kitāb al-Nafs*, and invariably dismisses any interpretation of Aristotle's epistemology that makes it empirical.

and its greatest importance lies in natural philosophy and in such related arts as medicine. Indeed, Avicenna's examples in this chapter are of physical causation, for instance, that 'the lodestone attracts iron' or that 'scammony purges yellow bile'.[21]

Ibn Sīnā has gone some way towards saving the letter of Aristotle's dictum that deprivation of sensation produces a deprivation of knowledge.[22] With a few exceptions (which are not mentioned here), people usually need sensory information to permit intellectual apprehension of species of existent things that are sensible in essence. They may need observations of sense to remind them of intelligible premisses not thoroughly acquired previously. Most significantly humans usually require repeated observation of natural things to produce 'empirical' laws, such as 'the lodestone attracts iron'.

The greater part of the necessary technical analysis has just been presented in connection with Avicenna's first discussion of knowledge and experience. His second account of these matters, in *Maqāla* III, *faṣl* 8 of *al-Burhān* need only be touched upon.[23] Let one point alone be stressed: in this chapter Ibn Sīnā is able to postpone the inevitable confrontation of Aristotle's views only by a deliberate but rather ingenious misinterpretation of what is said in the parallel chapter (I: 31) of the *Posterior Analytics*. There Aristotle talks of the effects produced by a lack of sensory data (literally, 'a failure of sense-perception', but the context is unusually limpid); Ibn Sīnā chooses to understand this as concerning the effects of an 'incapacity of sense to penetrate', for which there is no textual basis, Greek or Arabic.[24] Avicenna thereby allows himself to cover, rather more quickly, much of the same ground already traversed in chapter 5.

It is the concern of the intellect, he states, to devise from repeated particulars an intelligible abstract universal *(kulliyy mujarrad maʿqūl)*, an intelligible meaning to which sense has no access. Thus, for example, neither can one sense every eclipse nor can one sense any eclipse universally. Instead, Avicenna tells the reader once again, the intellect obtains the abstract universal by the light from a divine emanation. The intellect often 'snares' universal

21 *Ibid.*, p. 224, 1.2 The famous 'empirical method' (regarding the use of compound medicines) in Ibn Sīnā's *Canon of Medicine (Al-Qānūn fi'l-Tibb)*, is indeed 'empirical' in this sense. The discussion there holds virtually nothing of epistemological interest, however, and nothing at all for psychological theory. (See *Canon* II 1 2 and 3, Arabic text, Cairo (Būlāq), A.H. 1294 (1877), Vol I, pp. 224-231. Again one finds the example of scammony.)

22. See *Burhān*, Cairo ed., p. 224, 1. 11, where Avicenna ends his discussion by saying, 'Therefore, everyone deprived of a certain [amount of] sensation is deprived in respect of a certain [amount of] knowledge, even though sensation is not [itself] knowledge'. *Cf.* note 12, above.

23. *Ibid.*, pp. 249, 1. 11 - 250, 1. 10, esp. p. 250, 11. 1-6.

24. The 'misunderstanding' of Aristotle here is thoroughly treated in ʿAfīfī's introduction, *ibid.*, pp 39-40. The crucial line comes at *Post. An.* I· 31, 88a 11-12; in Mattā's Arabic translation, the phrase is *faqdu'l-ḥiss* (ed. ʿAbdul'l-Raḥmān Badawi, in *Manṭiq Arisṭū*, vol. II (Cairo, 1949), p. 398).

Although this degree of distortion in the use made of the inherited technical vocabulary by Ibn Sīnā is rare, it should be emphasized that the method as such is standard with him, and perhaps not much less so with Aristotle and most ancient and medieval philosophers. The philosophical and scientific usages of a term are analysed, and a meaning is then adopted which in part 'saves' the earlier ones but also reinterprets and refocuses them, so that the significance of the word is shifted and may be greatly distorted. (Perhaps the most amusing example in Ibn Sīnā is his blithe equation of the Galenists' terms for the higher psychological faculties with his own not dissimilar names, when he knows full well that his psychological schema is radically different from theirs and thoroughly anti-Galenistic. Many medieval and modern physicians and scholars have thus been misled. Similar remarks might possibly be made about his use of the language of the *ṣūfī*'s in the *Ishārāt)*. Potential converts to an unfamiliar intellectual position are to be won over, Avicenna's writings reveal, by the use of a familiar language which contains some suitably reinterpreted terminology.

Only the means designated as '*tajriba*', which, however, is the most important and interesting of the ways through which sensation can contribute to *taṣdīq*, now remains to be treated in *Burhān* III: 5.[19] In discussions relating to cognition, '*tajriba*', like '*empeiria*', means 'experiencing', 'gaining or having experience of' or '. . . acquaintance with' or '. . practice in', with a connotation of 'testing' or 'trying out' in the case of *tajriba*. Avicenna here describes *tajriba* simply as having in it 'a mixture of sensory "induction" *(istiqrā' ḥissī)* with intellectual deduction *(qiyās ʿaqlī)*'.[20] Aristotle's '*empeiria*' seems to have been a *hexis*, a 'developed state' of the soul, but Avicenna's '*tajriba*' looks at this point to be a process; on this, more below. In any event, *tajriba* is a judging through many particular examples that there exists a constant relationship between two universals such that a certain premiss asserted of them may be given assent. It seems reasonable to infer from Ibn Sīnā's abbreviated explanation that individual happenings gradually limn a universal, the representation of which is then completed by examining (or 'testing'?) further instances. One is actually told only that after sense-reports of often-repeated happenings of the same specific sort have been received, the intellect judges that the conjunctions involved are essential *(dhātī)*, not coincidental *(ittifāqī)*, because 'coincidence does not persist'. So the intellect is able to abstract what is in essence from what is by accident after a sufficient amount of 'experience'. In this manner *tajriba* will generate *taṣdīq*, according to the present account, and 'experience' will actually bring to pass ([w-q-'], II) in human minds proper universal cognitions.

'Experience' necessarily is concerned only in things accessible to sense,

19. *Ibid.*, pp. 223, l. 16—224, l. 5.
20. *Ibid.*, p. 224, ll. 6-7; *cf.* p. 223, l. 16.

it is not linked with *tajriba*. Indeed the sifting process is not granted a name, nor in this chapter are its products given any special designation.

When he turns to *taṣdīq*, Ibn Sīnā finds not one but four ways through which sensation can contribute.[16] The first is 'by accident' *(bi'l-ʿaraḍ)* where apprehension *(taṣawwur)* of one or more of the simple universals has been achieved with the help of the senses in the manner already explained, and the intelligibles have then been combined directly. *Taṣdīq* is here an immediate result of the 'light' of the Active Intellect; in Avicenna's words, this kind of intellectual assent occurs only 'through conjunction *(ittiṣāl)* of the [human] intellect with the light *(nūr)* from the Creator emanated upon souls and nature, which is called the Active Intellect (*ʿaql faʿʿāl*) and which is the agent that leads the [human] potential intellect out into act'.[17] It must be noted that the 'light' is only ultimately, not immediately, 'from the Creator', and that 'the Creator' designates the One or the Necessary Being of the philosophers, not the creator-God of the *Qur'ān* and the Bible.

The second way of reaching *taṣdīq* from sensory starting points is the 'particular syllogism' *(qiyās juz'ī)*. By this phrase Avicenna means a predicating about some natural species of something already known to be predicable of its proximate genus, through having apprehended by sense individuals which belong to that species (and *a fortiori* to the genus).

In the third place comes 'induction' *(istiqrā')*, a term which usually stood for the Greek word '*epagogē*'. Whereas Aristotle meant by *epagogē* an advancing from all available individual instances to a universal judgement, Ibn Sīnā perversely chooses to denote by *istiqrā'* a process in which the attention of the intellect is merely drawn to a relationship among universals by one or more perceptible examples of it, whether this be in the first instance or later on as a reminder. The intellect becomes aware of believing the intelligible relationship, but the 'induction' itself does not create that belief. By means of *istiqrā'* sense is only able to occasion the acceptance of premisses, and that almost trivially.[18]

For Avicenna, of course, the inductive leap in the usual sense is ontological as well as logical, so a metaphorical understanding of *epagōgē* is the best that can be expected. Even so, his interpretation of *istiqrā'* certainly must be called guileful, for it does not preserve the meaning that a reader of works of *falsafa* is justified in expecting. Its principal merit may be to obviate a later explanation of Aristotle's doctrine (100b3-5 in *Posterior Analytics* II: 19) that 'the method even by which sensation implants the universal in us is inductive'.

16. *Burhān*, Cairo ed., III:5, pp. 222, l. 17-224, l. 10
17. *Ibid.*, p. 223, ll. 3-4.
18. *Ibid.*, ll. 11-15, contains the description of *istiqrā'* Perhaps it is meant as a gesture towards Plato's *anamnēsis* – the main account, in the *Phaedo*, should have been known to Ibn Sīnā.

are either received completely and correctly by the rational faculty, since they are its proper objects, or are not received. When the simple intelligibles have been combined, connected, that is, in such a way as to be expressible in syllogistic premisses, the resulting composites may be either true or false; so beyond simply apprehending their intelligible content the mind must judge whether they are right, must gain conviction about their truth or falsity. The second stage, the accepting of the composite intelligible or premiss, Ibn Sīnā calls *taṣdīq*. This word was regularly used by Arab translators to render Aristotle's *pistis*, which was something logically different, being the confidence or conviction associated with the intellectual assent to a premiss. Nonetheless the usage of *taṣdīq* employed by Ibn Sīnā and the distinction between *taṣawwur* and *taṣdīq* are standard in Islamic philosophy.[15] The ideas of *taṣawwur* and *taṣdīq* and their relation to simple and composite objects of thought seem to depend ultimately on Aristotle's remarks about the subject, for example in *Metaphysics* IX: 10 and in *De Anima* III: 6, although there are perhaps also Stoic influences.

The accounts of the acquisition of knowledge given by Aristotle in *Posterior Analytics* II: 19 and *Metaphysics* I: 1 did not make full and consistent use of this analysis. Avicenna, however, is obliged by hindsight to do so. In the *Posterior Analytics* Aristotle was writing about the starting-points for *epistēmē*, so he should have concerned himself with the grasping of first premisses; but his description seems really to apply only to the separate universals contained in those premisses. In particular, *empeiria* emerges as the cognitive condition which results from the sifting and ordering of repeated evidence of the senses and which permits the rise of universal *concepts* in the soul. But in the discussion in the *Metaphysics* Aristotle clearly referred to composites and made *empeiria* the immediate source of the *premisses* in the arts and sciences.

So Avicenna has a good deal of room in which to manoeuvre, even if he wishes to be purely Peripatetic. His first move in *Burhān* III: 5, as was noted, is explicitly to restrict the possible range of empirical cognition to objects that are sensible of essence. He then separates his analysis of *taṣawwur* from that of *taṣdīq*, and for the present, limits his discussion of the function of *tajriba (i.e., 'empeiria')* to the second stage of the acquiring of intelligible premisses, to *taṣdīq*.

The sorting of the sensory contents of the soul in preparation for the *taṣawwur* of incomposite universals remains more or less as it was in Aristotle, but

15 The standard examination of this topic, no longer completely satisfactory, is Harry Austryn Wolfson, 'The Terms *Taṣawwur* and *Taṣdīq* in Arabic Philosophy, and their Greek, Latin, and Hebrew Equivalents', *The Moslem World* 33 (1943), pp. 1-15, repr. in Harry Austryn Wolfson, *Studies in the History of Philosophy and Religion*, vol. I, ed. I. Twersky and G. H. Williams (Cambridge, Mass., 1973), pp. 478-492. (See also Josef Van Ess, *Die Erkenntnislehre des Aḍudaddīn al-Īcī* (Wiesbaden, 1966), pp. 95-113; *passim*, and Fehmi Jadaane, *l'Influence du Stoïcisme sur la pensée musulmane* (Beirut, 1968; *Recherches... de l'Institut de Lettres Orientales de Beyrouth*, sér. 1, t. 41) pp. 106-113, *passim*.

truly can be said to attain to knowledge. And indeed, despite his lengthy discussion of the help provided by the senses, Ibn Sīnā does not deviate from this position even here in *Burhan* III: 5. Were he forced to summarize what he has actually asserted in this discussion he would be unable to save Aristotle's doctrine. He could come no closer than to claim that for people other than prophets and the best philosophers, sensation provides support that is widely necessary as an aid for intellection when they are first acquiring certain branches of learning, and that lack of sensation under those conditions does mean a loss of knowledge.

A résumé of Avicenna's description in this chapter of the psychological processes used in gaining knowledge of the temporal world will facilitate the tracing out of the developments that occur in his next two accounts. That the sensible and intelligible natures in things are distinct is his starting-point here: sense does not encounter the nature of man, for example, *qua* generalizable *(al-insān al-mushtarak fihi)*. The 'man' apprehended in the human intellect through the essential definition *(ḥadd)*[13] has been abstracted ([j-r-d], II) from all the accompaniments and individualizations of material existents, and *qua* abstract it is no object of sense. What the external senses do is merely to take up the sensible form and deliver it to the representative faculty *(khayāl)*, *i.e.*, to the sensory memory, where it becomes subject to operations superintended by the individual potential intellect. The intellect causes the images to be compared and, noting what is different, abstracts that which is common; thus it pares away the accidents and obtains the intelligible essence – but not from the images themselves.[14]

As Ibn Sīnā explains in many places, but not in this passage, the potential intellect after being thus prepared acquires the intelligible from a separate and eternal intellect-in-act, the Active Intellect, indeed, which has already been described. Nor can the human intellect store the universal thus gained; it is able only to increase the degree and range of its receptivity and remember where to 'look' for intelligibles previously possessed.

Up to this point Avicenna has been dealing with the apprehension *(taṣawwur)* of incomposite universals, which the mind either grasps or does not, which, in other words, are not true or false in themselves but in every case

13. *Taṣawwur* of the incomposite intelligibles is primarily by way of the *ḥadd*, see *Shifā': Ilāhiyyāt*, V: 5, 7, and 8, *passim*, and *cf.* III: 8 (The best text of Avicenna's *Metaphysics* is in the *Shifā'*, Cairo ed.: *Al-Ilāhiyyāt*, vol. I ed. by G.C. Anawati and Sa'īd Zā'id, vol. II ed. by Muḥammad Yūsuf Mūsā, Sulaymān Dunyā, and Sa'īd Zā'id (Cairo, 1960).) The *ḥadd* in this, its narrowest technical sense, is the abstract, intelligible nature *(ḥaqīqa)* of an *infima species*, which is also present in each individual of the given species and comes to it from the Active Intellect as *dator formarum* (*cf.* *Ilāhiyyāt* IX: 5, *passim*). The *ḥadd* when expressed as the formulable essence of a species becomes its essential definition, still called the '*ḥadd*' (now strictly = Gk. *horos* or *horismos*). This idea of the rôles of the *ḥadd* is a comparatively obvious extension of Aristotelian teaching; *cf.*, especially, *Meta.* VII: 4, 1030a 2-17.

14. *Burhān*, Cairo ed., III:5, pp. 220, l. 8-222, l. 16.

or used in imagination, are derived from sensations, Ibn Sīnā tells his readers, and with such images the human intellective faculty can act in such a way as to acquire incomposite universals. These it can then join together into definitions, premisses, and syllogisms. Sensation in this way is a principle for the apprehension *(taṣawwur)* of intelligible universals, but only by accident *(bi'l-ʿaraḍ)*, not in essence *(bi'l-dhāt)*. In the sciences concerned with things that have corporeal existence, and are thereby sensible of essence, that same division of function between sensory and intellective processes is to be found also in the acquiring of primary *premisses*, *i.e.*, those from which demonstration has its start; sensation plays a part in the recognizing of first premisses (provided they relate to things sensible) as well as in apprehending the universal terms they contain and the subsequent middle terms that are needed to construct the demonstrations. In other words, the products of sense-perception are a source for the objects of *nous*, in the narrower Aristotelian sense of 'direct intellectual grasping', whether they be incomposite or composite. Sensory processes may also be employed, it turns out, in testing derivative premisses, empirically.

But sensation will ultimately be allowed only as a basis, and often a dispensable one, for acquiring the genuine universals. Even in this early chapter one discovers that things which in their existence are sufficiently unconnected with matter as to be *intelligible* in essence cannot be apprehended from any sort of sensory foundation. Some few persons, moreover, have strong enough intellectual faculties, Avicenna maintains, that they can attract all or most intelligibles without recourse to information from the senses; other persons less gifted but still intellectually able can develop their intellects to a level where reference to sense-data and imaginings becomes unnecessary. These doctrines, which are not developed in the *Burhân*, appear in the *Kitāb al-Nafs* and elsewhere; furthermore, it is safe to infer from discussions in the *Kitāb al-Nafs* and the *Ilahiyyat* that all persons can obtain at least a few of the universals that relate to the natural world without any recourse to the senses or to imagination.[12a] Since, finally, it is only the intellect, when complemented from without, that can grasp the pure universals, only the intellect

remark by Aristotle about a loss of sensation, *Post. An.* I: 18, 81a 38-40, is repeated by Avicenna at p. 220, ll 5-7, in the present chapter (and *cf.* p. 224, l. 11).

See also ʿAfīfī's description of the correspondences between Avicenna's and Aristotle's texts, pp. 36-37 in his very useful introduction to this work.

(At *De Anima* III: 3, 432a 7-10, and *De Mem. et Rem.*, 1, 439b 31 *sqq.*, Aristotle makes a related claim, that if one perceives nothing though the senses, one is incapable of learning anything).

12a That in principle every corporeal aid to human intellectual cognition is dispensable is something Avicenna seems never to assert outright. It is necessary to study all the possibilities one by one to extract the generalization, which remains provisional, even though any exceptions will have to have a narrow range. See, *int. al.*, in the *Shifā'*, *al-Ilāhiyyāt* III, 8 and IX 7 and *Kitāb al-Nafs* V 3 (with care) and V 5 and 6, as well as some relevant passages in *Al Risala al-Adhawiyya*. Note especially *Kitāb al-Nafs* V: 6, pp 248-250, *ed. cit.* in note 8 above, English translation in Rahman, tr., *op. cit.* in note 10 above, pp. 35-37 (= pp. 166-168 of the Arabic text of the *Najāt*, *ed. cit.* in the same note).

cognizable objects that are more abstract and less immattered, quasi-universals like the lower kind of things that are now called 'intuitions'. These products of the *wahm*, which Ibn Sīnā designates *ma'ānī*, are perhaps best referred to by the Scholastic term '*intentiones*'. A stock example is the intuition of 'enmity' that a sheep forms about wolves; although post-sensational, it is not completely abstract, not 'intelligible'.[11] For a person, *intentiones* are the final and most abstract result of his apprehension of the sensory world. They provide the nearest Avicennian equivalent to what Aristotle called '*empeiria*' when he spoke of 'experience' arising from repeated memories of the same thing (*cf. Metaphysics* I: 1, 980b 25-981a12, and *Posterior Analytics* II:19, 100a3-9). These *intentiones* can show a person's intellect where to 'look' in the intelligible world for the true universals – the concepts and ideas contained in the indemonstrable first premisses and subsequent middle terms which build up the demonstrative sciences. But knowledge as such arises solely through intellection: through grasping the intelligibles, which emanate into human minds only from the separate Active Intellect, in which also they are stored. In this way Ibn Sīnā has found a rôle for the senses and for experience in reaching knowledge, but knowledge itself has been kept absolutely intellectual and incorporeal, essentially independent of sensation and everything bodily.

The main account of this borderland between psychological theory and epistemology comes, as one would expect, in that book in the logical *jumla* of the *Shifā'* which corresponds to Aristotle's *Posterior Analytics*, *viz.*, the [*Kitāb al-*] *Burhān*. As might also be anticipated, the treatment is not straightforward. One must look at three fairly widely separated chapters, III: 5, III: 8, and IV: 10, and cope with a lack of candour concerning Aristotle's views that ranges from mild deviousness to intentional and unblushing misrepresentation. One is taught a great deal, however, about how Ibn Sīnā expounds and develops his ideas – a sobering and cautionary experience for anyone tempted to use the *obiter dicta* of Avicenna as a basis for construing his doctrines.

In his first discussion, the one in [*Kitāb*] *al-Burhān* III: 5, Avicenna constructs an interpretation of the subject-matter of *Posterior Analytics* I:18 and tries to show that loss of sensation results in loss of knowledge, as Aristotle there has clearly stated.[12] Images in the soul, including those stored in memory

11. The main treatment of the *wahm* is located in the *Kitāb al-Nafs* of the *Shifā'*, Bk. IV, chs. 1 and 3 (*ed. cit.* in note 8 above, pp. 163-169 and 182-194); other discussions are to be found in I. 5, III: 8, and the last part of V: 6 (esp. pp. 45-46, 153-154, and 244-246). The connection with *tajriba* is mentioned in IV: 3, pp. 182-185.

The *Najāt* again presents a rudimentary but helpful summary of the doctrines. See Rahman, tr., *op. cit.* in note 10 above, pp. 30-31 in ch. 3 and pp. 39-40 in ch. 7 (but ignore the commentary, which here no longer stands up well).

12. *Shifā'*, Cairo ed., *Al-Manṭiq*, 5: *al-Burhān*, crit. ed. and introd. by Abu'l-ʿAlā' ʿAfīfī (Cairo, 1956) (hereafter, '*Burhān*, Cairo ed.'), *Maqāla* III, *faṣl* 5, pp. 220-227. Only pp. 220-224, l. 11 are relevant here. Aristotle is not mentioned by name, Ibn Sīnā merely writes 'qīla ...', 'it has been said...'. The

eternal world that is grasped by the intellect. (In this, of course, Ibn Sīnā follows an ancient Greek intellectual tradition that goes back at least to Parmenides). These realms never overlap, and they meet only in the human species, in each individual soul. There, the lower world rises as far as sense-perceptions *(maḥsūsāt)* and 'estimative' *intentiones* (see below), and the intelligible world reaches down to the potential intellect, which it renders actual. Sensibles *(maḥsūsāt)* – sensory information of any kind – do not contain, and sensation cannot grasp, any true universals *(kulliyyāt)*. Consequently, Ibn Sīnā may not allow any genuinely empirical theory of the acquisition of knowledge: in the end, authentic knowledge *(ʿilm)* can be attained by a human being only through his externally actualized intellect *(ʿaql)*.[10]

Induction *(istiqrāʾ*: translates Greek *epagogē*), in particular, is strictly if disingenuously proscribed as a generative source of knowledge. But when it comes to *empeiria* (rendered in the Arabic texts as *tajriba)*, Avicenna equivocates, for he is anxious to save Aristotle's all-too-unambiguous presentations of the empirical basis of knowledge in *Metaphysics* I: 1 and, especially, in *Posterior Analytics* II: 19. Experience, Ibn Sīnā decides, can *lead* to knowledge; and, tortured also by Aristotle's plain speaking in *Posterior Analytics* I: 18, he even grants that sensation may be regarded as a principle of knowledge – but, the reader can infer, not as a strictly essential *(dhātī)* principle nor by any means as a sufficient one.

The connection between *tajriba* and *ʿilm* is eventually explained in terms of two faculties that seem to be among Ibn Sīnā's own contributions to the analysis of the soul, the 'estimative' faculty *(wahm, quwwa wahmiyya)* and the 'storehouse' or special memory associated with it, which is called the retentive, or memorative, faculty *(ḥāfiẓa; dhākira)*. From the sensible images contained in the soul, whether they are simply remembered or have been separated and recombined in imagination, the estimative faculty forms

10. Besides the main reference given on p. 52, above, see also the preliminaries contained in *Kitāb al-Nafs*, Bk. I, ch. 1 (last third); IV: 2 *(passim)*, V: 1 (second half), and V: 2 *(passim)* (*ed. cit.* in note 8 above, pp. 12-16, 163-169; 204-209, and 209-221). Short discussions pertinent to the question of *tajriba* and *ʿilm*, both subordinate to the main accounts in the [*Kitāb*] *al-Burhān* (for which see below), appear in Bk. II, ch. 2, of the *Kitāb al-Nafs* and in V: 3 (*ed. cit.*, pp. 60-61 and 221-222).

An incomplete presentation of the psychological theory of the acquisition of knowledge is to be found in the *Najāt*, see F. Rahman, tr., *Avicenna's Psychology. An English Translation of 'Kitāb al-Najāt'* .. (London, 1952), chs. 5, 7, and 11, pp. 33-35, 40, and 55 (corresponding to pp. 165-166, 170-171, and 182 of the second edition (Cairo, 1938) of the Arabic text), for *tajriba*, see esp. p. 55.

Chapter 16 of this part of the *Najāt* (Rahman, tr., pp. 68-69, Arabic text, *ed. cit.*, pp. 192-198) is also relevant, although unlike the other chapters mentioned it has not actually been excerpted from the *Shifāʾ*. The full doctrine is simplified here by omitting the rôle of 'estimation'; compare the remarks below on Ibn Sīnā's similar procedure in the [*Kitāb*] *al-Burhān*.

Persons unfamiliar with the area of thought to which Ibn Sīnā's psychology belongs may be helped by the rather advanced introduction to be had in Herbert A. Davidson's 'Alfarabi and Avicenna on the Active Intellect', *Viator* 3(1972), 109-178.

In the present instance, Avicenna's elaboration of the Aristotelian view has required two entities from without, instead of only one, to complete each human soul. Aristotle's ambiguities were more economically resolved by Alexander of Aphrodisias and by Themistius, and will be so done again by Averroes. But these departures from Aristotle permit Ibn Sīnā to save his non-Peripatetic conceptions of immortality and of intellection. It is not too strong to say that his own peculiar idea of personal salvation determines the nature of his solution to the problem of abstract (*i.e.*, intellectual) thought and, derivatively, to the ensouling of the embryo.

The main difficulty that Avicenna has to face is accounting for the individuation of an *intellect*; nor does he ever satisfactorily explain it. His approach to the question depends upon the materially individuated potential intellect, which is not problematical in this respect. He attaches the potential intellect to, or identifies it with, a person's rational soul, which he has made the 'intellect' that enters the embryo 'from without'. But the continuing individuality of the potential intellect when conjoined to the Active Intellect is left unexplained. *Qua* individual, an intellect must be attached to a body and therefore be mortal; *qua* actual and eternal it should not be individual. (Whether certain aspects of the rational soul as presented by Ibn Sīnā justify recent talk of an 'ego'-concept, or something similar, in his psychology, and whether, if so, that would help solve the problem of individual intellects is a question that I shall take up briefly at the end of the paper.) Ibn Sīnā's aim, in any event, is to justify a scheme whereby the individual potential intellect perfects itself by continually rising to the grade of 'acquired intellect' and receiving actual intelligibles from the separate Active Intellect, so that it can function continuously and *in actu* after the body has died. The Active Intellect, moreover, with its eternal, actually intelligible contents, remains in Avicenna's program safely outside the corruptible human realm.

But however pleasant this knitting together of psychological, embryological, and soteriological doctrines may be,[9] it is only byplay to the main philosophical drama that derives from Ibn Sīnā's conception of immortality. The centre of the action lies in his metaphysics: in epistemology first and then, without resolution, in ontology.

By way of preface to the second, epistemological example of the influence of Ibn Sīnā's psychological theories, it is necessary to emphasize the radical distinction in Avicennian metaphysics between the corporeal and corruptible world that is apprehended by the senses and the higher, immaterial, and

9. Of course the synthesis is *not* pleasing insofar as it multiplies entities. All too often Avicenna systematizes by merely adding theories together, however well he finishes the joins his thought never becomes a perfectly unitary structure, for he attempts to incorporate too much.

Even so, as will become evident, a great deal of his conciliatory discourse is aimed at disarming criticism of what is actually rigorous and proper system-building on his part.

In Avicenna's *Ḥayawān* it is the rational soul that corresponds to Aristotle's intellect 'from without', and this is the human intellectual faculty as such, the undeveloped capacity for receiving intelligibles. For Aristotle, however, one may reasonably conclude that the intellect 'from without' was of the self-sufficient kind which seems to have been implied by his description in *Posterior Analytics* II: 19 of how universals are acquired, and which therefore must include both the passive and active intellectual faculties that have so tantalized the interpreters of *De Anima* III: 5. But howsoever one chooses to resolve the ambiguities of Aristotle, there are none left here in the *Shifā'*. The intellect 'from without' of the *De Generatione Animalium* has become the rational soul, which is an intellect *in potentia (bi'l-quwwa)* (and which originates from the Active Intellect, in this entity's rôle as *dator formarum*; *cf. al-Shifā', al-Ilāhiyyāt* IX: 5). On the other hand, the active human intellect, in accordance with an exegetical tradition descending from Alexander of Aphrodisias (*fl.* early 3rd cent. A.D.), has been made external and is, indeed, one aspect of the Active Intellect. The human intellect-in-act, however, is now interpreted as the individual's 'acquired intellect', produced through the 'illumination' of his passive intellectual faculty by the true Active Intellect; it has thus become the mere effect of *another* action 'from without', a collection of intelligibles lent from above. In whatever way Aristotle is to be understood, it is quite certain that he wished to have only *one* entity from without involved in the human soul, but Avicenna, with Muslim largesse, has given us two.

There is no excuse for considering this a mistake on the part of Ibn Sīnā. He is not explicating the texts of Aristotle, but is expounding a consistent philosophy of his own within the general confines of Islamic Peripateticism. That one may speak of a correspondence between chapters of the *Shifā'* and chapters of Aristotle's works only reflects the fact that the *Shifā'* is an encyclopaedic work covering the whole of Greek philosophy (in the first, second, and fourth *jumlāt)* and the mathematical sciences (in the third *jumla)*, whose basic order of exposition in logic, natural philosophy, and (to some extent) metaphysics follows the standard Arabic arrangement of the Aristotelian *corpus*. (Material equivalent to certain other works, such as Porphyry's *Eisagoge*, is added in; and in the *Metaphysics (al-Ilāhiyyāt)* are included various further topics, owed mainly to al-Farābī, that are ethical, political, or religious in nature and replace the 'theoretical' content of the standard texts in ethics and politics (of which Aristotle's *Nicomachean Ethics* and Plato's *Republic* are the most important). It may be assumed that the *Shifā'* is intended to be read by serious students in place of the books by the Greek authors). Consequently, the *Shifā'* often takes over the structure of Aristotle's writings, sometimes down even to the sequence of thought within individual paragraphs. But its views are as independent of Aristotle's teachings as Ibn Sīnā feels to be desirable. The next example will make this assertion more obvious still.

First, however, the biological matters. What specific changes does Avicenna's theory of immortality generate in Peripatetic teachings about the ensoulment of the human embryo? The problem is set by Aristotle's notorious discussion in *De Generatione Animalium* II: 3, where he speaks of the intellect *(nous)* 'from without' *(thurathen)*. Ibn Sīnā's treatment comes in the *Shifā'*, *al-Ḥayawān* XVI: 1;[5] his account at the start follows Aristotle, but it ends with a notable addition.

The vegetative level of the soul, which oversees the development and growth of the embryo, is received with the semen of the father, Ibn Sīnā asserts; and in the semen there is also something which is 'prepared to receive the connection *(ʿalāqa)* with the soul', *viz.*, the (vital) heat, which is not fiery like elemental fire but is analogous, rather, to the heat which emanates *(yafīḍu)* from the heavenly bodies and is ultimately related to their substance *(jawhar)*.[6] So far, reasonably orthodox Aristotelianism. Also, Avicenna says, when the heart and the brain have come to exist in the embryo, the sensitive *(ḥissiyya)* soul emanates *(tafīḍu)* from the vegetative, and the rational *(nuṭqiyya)* soul becomes attached to it (to the vegetative organism, apparently, at the same time as the sensitive soul is produced). Still Aristotelian, although everything has been consolidated in such a way as to permit the highly tendentious constructions which now follow. The rational soul, Avicenna continues, is different from the other two levels and has nothing do to with matter as a substrate, but the soul (*qua* rational) is not yet effective *(ʿāmila)*, being like that of the drunk or the epileptic. It is completed ([k-m-l], X) only by something external, when, in the person's childhood, that entity first assists the intellect *(ʿaql)* *(i.e.*, enables it actually to think).[7]

The last statement may be made more explicit by reference to the *Shifā'*, *Kitāb al-Nafs*, especially V: 5 and 6. The rational soul which enters the embryo has but the bare potentiality for intellection, the grade of intellect that Ibn Sīnā calls 'material' *(hayūlani)*. This potentiality becomes actualized, becomes truly an intellect, by receiving intelligibles as such from the separate and eternally actual Active Intellect (*ʿaql faʿʿāl)*, the lowest of the celestial intellects. The grade of its potentiality increases by degrees, but the rational soul attains the intellect *in actu (bi'l-fiʿl)* only when it is 'borrowing', or 'has acquired', actual intelligibles from the Active Intellect. It then possesses a true intellect, called the 'acquired' *(mustafād)*, which is correctly the second entity referred to above, the intellect which 'completes' or 'perfects' the rational soul.[8]

5. Ibn Sīnā, *Al-Shifā'*, ed.-in-chief Ibrāhīm Madkūr (Cairo, 1952-), hereafter referred to as '*Shifā'*', Cairo ed., *Al-Ṭabīʿiyyāt 8*: *al-Ḥayawān* [more properly, '*Fī Ṭabā'iʿ al-Ḥayawān*], ed. ʿAbdul-Ḥalīm Muntaṣir, Saʿīd Zā'id, and ʿAbdullāh Ismāʿīl (Cairo, 1970), *Maqāla* 16, *faṣl* 1.

6. *Ibid.*, p. 403, ll. 1-3 and 8-11.

7. *Ibid.*, ll. 3-8.

8. These chapters contain the main exposition of Ibn Sīnā's theory of the acquisition of knowledge through the intellect and its subsidiary faculties, see F. Rahman, ed., *Avicenna's 'De Anima'* [*Al-Shifā': Kitāb al-Nafs*] (London, 1959), pp. 234-250.

le'), and, in some areas, from writings in the Galenic tradition. Specifically, Ibn Sīnā's psychology in both approach and content was principally Aristotelian. There had been incorporated within it, however, certain insights that belonged ultimately to Plotinian philosophy; and there had also been accomplished the more difficult and less precedented task of transplanting into it certain 'religious' conceptions, Muslim in Avicenna's own eyes but scarcely so in most others.

Ibn Sīnā's idiosyncratic notion of individual immortality required an elaborate and painstaking integration into his philosophy, into psychological theory first and then into related areas throughout the system. Biology, epistemology, ontology, ethics, and political science, each and all needed to be modified. The idea itself which Ibn Sīnā had formed of personal salvation was simply that an individual's intellect could be developed during the person's lifetime to the point that it would survive the death of his body and become a part, still self-identical, of a celestial intellect.[4] Thus a human being of sound mind would have as his chief task in life the full actualization of his mental capacities, so that deprivation of his senses, his imagination, and his estimative faculty would leave him still able to think. An intellect fully developed in this way would not perish with the body, and the person's resurrection *(maʿad)* into paradise would amount to entering a self-conscious but bodyless state of eternal intellection-in-act. One would have reached the intelligible world contained in the lowest of the celestial intellects. This explanation was Avicenna's own, although it had something of the spirit of Plotinus and of al-Fārābī. It was thoroughly non-Aristotelian, and thus proved to be anathema not only to ordinary Muslims but also to pure Peripatetics such as Ibn Rushd.

The requirements of this sort of immortality greatly influenced Ibn Sīnā's theories. The two doctrines examined in the remainder of this paper both show its effect. In the first, a fairly straightforward modification was made to Aristotelian embryology. In the second instance, a radically anti-Aristotelian epistemological doctrine was adopted; neo-Platonic in appearance but unlikely so in inspiration, it lay at the heart of Ibn Sīnā's psychology and metaphysics. Both tenets were at root, I believe, philosophical responses to the Muslim precept of personal salvation; and they both had a place in the 'dialogue' between the *falāsifa* and the other groups of Muslim intellectuals. Indeed in the latter case, where Avicenna effectively denied the necessity and, rigorously speaking, even the possibility, of acquiring knowledge from experience, the doctrine should be considered one of the most important contentions in that debate as regards the consequences for philosophy and the other Greek sciences.

4. This non-Qur'ānic view is only adumbrated in the *Shifā'* (*Kitāb al-Nafs* V: 5 and *Ilāhiyyāt* IX: 7 and X: 1), the complete, 'esoteric' teachings are presented in *Al-Risāla al-Adḥawiyya fi'l-Maʿād* (ed. with Italian tr., introd. and notes by Francesca Lucchetta, as *Epistola sulla vita futura*, vol. I (Padua: Antenore, 1969)).

thought during that period. Let me note in this connection, without undue emphasis, that the title of the *Kitāb al-Shifā'* is to be translated as 'The Book of the Healing [of the Soul]' and the name of the compendium of that work, the *Kitāb al-Najat*, as 'The Book of the Salvation [of the Soul]'!

The first of the two particular problems that I have chosen to investigate illustrates the integration of a religiously motivated psychological doctrine into a different area of philosophy, in this case embryology, in order to render it more acceptably Islamic. The second and more important example, the question of the empirical basis of knowledge, is intended to exhibit the significance of psychological theory for the career of Islamic science in all four of the ways that I have just described – explicitly as regards the general question of knowledge, and implicitly, but I trust plainly, with respect to the other three. The second example, moreover, should isolate the part which was played by Avicenna's own psychological thought; and it should make clear a major way in which Ibn Sīnā's theories in psychology, acting through his philosophy as a whole, led towards a transformation of the Islamic philosophical tradition while coordinating it more closely with its Muslim surroundings.

Both problems will illuminate the relationship of Ibn Sīnā to his Greek authorities, and the second will have a certain bearing on the vexed question of his 'mysticism'. The latter case, finally, will expose a serious but often unrecognized hazard that one frequently encounters when trying to determine Ibn Sīnā's true position on some issue – a difficulty arising from his method of presentation, even in his most straightforward discussions. Incomplete, especially adumbrative, exposition rather than tentative or shifting views will turn out to be his vice.

The examinations below of the zoological and the epistemological topics will both follow the *Shifā'*. It is this work, indeed, which nearly always contains Avicenna's basic account of his doctrines, even though in certain cases the explanation there is disingenuous or incomplete, and a franker or more developed treatment must be sought elsewhere. In the present instances, certainly, the *Shifā'* appears to need no important corrections.

The two Avicennian doctrines which are about to be considered were both consequences of the same basic Muslim belief, the idea of individual immortality and salvation. This tenet, stripped by Ibn Sīnā of any notion of bodily resurrection (at least in his franker, or more esoteric, writings), was a cornerstone of his psychological and metaphysical thought. The framework of theory into which it had had to be placed, *i.e.*, Avicenna's philosophical system as a whole, belonged in its methods and concepts to Greek philosophy in its Islamic guise, especially in the form it had taken at the hands of al-Fārābī (*ca.* 870-950). The teachings derived chiefly from Aristotle and occasionally from the Greek commentators, but they were also tinted – or tarred – with ideas from certain neo-Platonic works (including the pseudonymous 'Theology of Aristot-

themselves in a dialogue principally about philosophy and Qur'ānic religion, both among themselves and against the other interpreters of Islam. What I said earlier of the discussion in general is especially applicable here, namely, that a cluster of psychological issues assumed exceptional importance. The older questions of the definition of a believer, the nature of God's attributes, the createdness of the *Qur'ān* (largely replaced as a problem for the *falāsifa* by the createdness of the *world*), and freedom of the will had all been given stereotyped sets of answers by the tenth century. But other issues arose and demanded resolution: the nature of the soul; the distinctive characteristics of revelation, inspiration, dreaming, prayer, and ritual worship; the identifying criteria of true Prophethood (and thus of the basis for the Law – a vital matter for Islam); the personal immortality of individual souls, the manner of their salvation, and the nature of their bliss; the resurrection of the body, which was a prominent Muslim belief, but one that remained inexplicable within the limits of *falsafa*; the means whereby God can know particulars (and thus reward and punish individual believers properly, carrying out the 'Promise' and the 'Threat' of the *Qur'ān*); and the right mode and criteria of human knowledge. With the one significant exception of the eternity of the world, the major doctrinal problems that were set for the students of the ancient sciences by their Muslim environment required solution within one or another area of psychological theory. For Ibn Sīnā even political science reduced to an exercise in faculty psychology: the 'virtuous city'(*i.e.*, the best political community) was conceived as a society ruled through a Law that had been revealed by a true prophet, and the true prophet he identified as a man whose soul had a special faculty, an extra, higher degree of intellect called the 'prophetic' or 'holy' intellect, and who, through the overflow from this powerful intellect into his imaginative faculty, could put into images that were suitable for the common people all the essential conceptions of the Law and the religion.

The history of philosophy and science in Islam, then, was very greatly affected by the development of psychological theory in several ways: through the transformation in the nature of philosophy, through the changing ideas of the purposes of an intellectual life, through the framing of doctrines concerning the origin of knowledge, and most basically through the handling of contentious issues in the philosophers' general debate against other intellectual groups. Psychological questions were crucially involved in the processes that shaped classical Islamic culture, and theorization about the soul and its functioning thus shared indirectly but decisively in fixing the destiny of Islamic science.

I hope I have sketched enough background to make my initial assertions more plausible and persuasive. What I can do now is only to paint in a very small bit of the foreground. I shall discuss certain aspects of Ibn Sīnā's psychology in order to show the pervasive influence it had in his philosophy and to reveal at the same time the importance of psychological issues in Islamic

emphasis in philosophy away from cumulative investigation of the human and natural world towards metaphysical illumination (a change where the centrality of psychological questions has already been asserted) was in the end a metamorphosis whereby speculative philosophy effectively distanced itself from the several scientific disciplines and left them more susceptible to theoretical stagnation and to futile elaboration of a positivistic sort.

Secondly, the philosophers convinced themselves that the highest philosophic and human good and the greatest happiness *(saʿāda)* was conjunction *(ittiṣāl)* with a higher intellect. By so doing they very largely reduced moral philosophy to theoretical psychology, to discussion of this psychological state of quasi-union and the means of achieving it. Such a goal for the philosophical life would have appeared to most non-philosophers to be just a poor substitute for *ittiḥād*, the uniting with God depicted by the *ṣūfī*'s, and this view must have had a considerable effect in channelling the interest of educated Muslim youths away from philosophy itself and all that much farther away from the scientific disciplines.

In the third place, classical Islam was characterized by the extraordinary prominence granted by the entire society to the question of knowledge *(ʿilm)* -- of the kind of knowledge that a Muslim ought to accept as right and of the basis for certainty in that knowledge.[3] But asking what knowledge is, in effect implied asking how knowledge is to be obtained; and that meant understanding the operations of the soul. Greek philosophy and science, suitably modified, formed one way of knowledge that was open to the Muslim believer. Apologists of the Greek sciences *(al-ʿulūm al-awā'il)* were forced by the internal constraints of philosophical theorizing and the external demands of legitimization in Muslim society, to explain the special nature of their sort of knowledge and the basis of its claims to truth. The burden of these explanations fell upon psychological theory. But the account that was produced, *viz*., human participation in a higher intellectual world, left philosophy without a 'religious' justification as convincing as that of the Qur'ānically based disciplines or *ṣūfī* mysticism and without any good 'secular' substitute, such as, for example, a rigorous Aristotelian empiricism might have supplied.

Finally and most generally, psychology influenced the development of Islamic science by its assumption of the leading rôle when in the tenth century the students of *falsafa* and the other Greek disciplines attempted to come to terms with their Islamic environment. This they did by entering into the general debate which I mentioned, where each of the several opposed groups of Muslim intellectuals represented a different attitude to the religion and a different approach to knowledge. The adherents of the Greek sciences engaged

3 See Franz Rosenthal, *Knowledge Triumphant: the Concept of Knowledge in Medieval Islam* (Leiden: Brill, 1970); especially pp. 1-4 where something of Rosenthal's analytical framework is disclosed.

Naṣīr al-Dīn al-Ṭūsī (1201-1274) was able to effect through his exegesis of Ibn Sīnā in the *Sharḥ al-Ishārāt* and elsewhere. The philosophical *cursus*, which in the ninth and tenth centuries comprised logic and mathematics, natural philosophy and the mathematicized natural sciences, metaphysics, and ethics and politics, retained with Avicenna something of the original Aristotelian regard for research and the cumulative development of knowledge. Afterwards, however, it became a mere propaedeutic, albeit an essential one, for a directly illuminative, and supposedly more valuable, kind of knowledge, eventually interpreted in the later Iranian school as mystical gnosis. Although I cannot discover a real mysticism present in Ibn Sīnā's works, and certainly not in the frequently cited chapter on 'The Stages of Those Who [Seek to] Know' *(Maqamāt al-ʿĀrifīn)* in the *Kitāb al-Ishārāt (ed. cit.*, pp. 198-207), nevertheless illuminationist features were decidedly prominent, and the ground for the fully mystical development was thoroughly prepared by Avicenna's philosophy.

The driving force behind this transformation of *falsafa* derived, I am convinced, from the philosophical investigation of the soul, or rather from the implications that psychological doctrines yielded in nearly all areas of philosophical enquiry. The same ultimately psychological issues were also present to the *mutakallimūn* (the so-called 'rational theologians' of Islam) and the intellectually inclined among the *ṣūfī*'s – and indeed to all educated Muslims of those centuries. In the development of classical Islamic thought the primary task was the broadening and theoretical deepening of the Qurʾānically based religious culture. So it is not surprising that there was a scarcely interrupted general debate among opposed groupings of Muslim intellectuals – fundamentalist jurists, rationalist theologians, philosophers, *ṣūfī*'s, and Ismāʿīlīs, among others which had a great directive influence on the culture, and which very often addressed itself to matters in psychology. The question of the soul and the problems of right knowledge and right belief that were inseparably joined to it became and remained a fundamental concern, perhaps the most basic one of all, to Islamic thinkers. Psychological issues formed a vortex that eventually drew every theoretical system into its whorls, and usually threw it out again a shivered wreck. To understand the cultural history of medieval Islam it is essential to study the theories of the soul.

In a single paper one cannot document nor even illustrate all features of the description that has just been offered. But it seems appropriate to provide some indication of how psychological theories in the Islamic world had such importance specifically for the history of science.[2] In brief, I find that there were four ways, all of them indirect. (Of course there was a direct way, too, for psychology after all had long been a part of 'science'!) First, the shift of

2. These remarks were originally made in answer to a question asked from the floor by Prof A.I. Sabra They are incorporated here in their natural place in the text.

A Decisive Example of the Influence of Psychological Doctrines in Islamic Science and Culture:

Some Relationships between Ibn Sīnā's Psychology, Other Branches of His Thought, and Islamic Teachings

ROBERT E. HALL*

PSYCHOLOGICAL THEORY was a central concern of the medieval Islamic world, and Ibn Sīnā[1] was a key figure in the history of Islamic thought. Appropriately enough then, psychology was a main focus of Ibn Sīnā's own work, and his theories were of great importance in the history of psychology. Indeed, during the Middle Ages in Islam or in the West and, I am tempted to add, in the Renaissance, Ibn Sīnā was rivalled as a psychological theorist only by Ibn Rushd (Averroes; A.D. 1126-1198). But if I am right in my thinking, Ibn Sina's psychology had a further significance in Islamic intellectual history: for much of Ibn Sīnā's thinking revolved around the analysis of psychological issues; the philosophical system that he created signalled a turning-point in the history of philosophy and science and theoretical enquiry as a whole – even religious enquiry – in the Islamic world. So a correct grasp of Ibn Sīnā's psychological doctrines is prerequisite, I believe, to any full analysis of Islamic intellectual history and, *a fortiori*, to a proper understanding of the course of Islamic science.

Ibn Sīnā's *Shifā'* was the longest systematic exposition of *falsafa* (by which I mean simply Islamic philosophy in the Greek tradition) to have been produced in the classical period. Yet in the *Shifā'* and in Avicenna's other philosophical works was contained potentially (and even actually, in the view of certain present-day scholars) a radical transformation of the Islamic philosophical tradition: witness the abuse that Ibn Rushd heaped upon Ibn Sīnā for abandoning pure Peripateticism and the strikingly mystical philosophy which

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1. Ibn Sīnā (A.D. 980-1037) is the great physician and philosopher known in the West as Avicenna.

gold which is the goal of human life and which allows man to play the role for which he is destined, to act as the bridge between heaven and earth, as the eye through which God views His creation, as the channel through which the grace of heaven penetrates the earth and fecundates it. Through this inner alchemy, to which all other aspects of alchemy are subservient, man comes to see nature not as the chaos of coagulated matter but as the theophany which reveals the paradise which is here and now and which man must rediscover through the attainment of the gold which resides at the heart of all beings and which remains to be extracted by means which tradition offers to those who are willing to surrender themselves to it. Although Rāzī sowed the seeds of what was to become known later as the science of chemistry, Islam continued to harbor that spiritual alchemy which refuses to see nature as deprived of life, which aims at transmuting the inner being of man and attempts to bring about, through his transmutation, the spiritual revival of nature

Applied to nature, *ta'wil* means penetrating the phenomena of nature to discover the noumena which they veil. It means a transformation of fact into symbol and a vision of nature, not as that which veils the spiritual world, but as that which reveals it.

Alchemy is precisely such a science, one based on the appearances of nature, particularly the mineral kingdom, not as facts in themselves but as symbols of higher levels of existence. It is not accidental that Jābir was both a Sufi and also a Shīʿite and that in fact the Jābirian corpus later became closely associated with Ismāʿīlism which added certain treatises to the original body of Jābir's works.

Jābir, while also interested in natural occurrences, never divorced the facts of the natural world from their symbolic and spiritual content. His famous Balance (*mīzān*) was not an attempt to quantify the study of nature in the modern sense but "to measure the tendency of the World Soul". His preoccupation with numerical and alphabetical symbolism, with the study of natural phenomena as determinations of the World Soul, with specifically alchemical symbols, all indicated that Jābir was applying the process of *ta'wil* to nature in order to understand its inner meaning.

Rāzī, by rejecting prophecy and the process of *ta'wil* which depends upon it, also rejected the application of this method to the study of nature. In so doing, he transformed the alchemy of Jābir into chemistry. That is not to say that he stopped using alchemical terminology or ideas, but in his perspective, there was no longer any Balance to measure the tendency of the World Soul, nor any symbols to serve as a bridge between the phenomenal and noumenal worlds. The facts of nature were studied as before, but as facts, not symbols. Alchemy was studied, not as real alchemy, but as an embryonic chemistry. The religious and philosophical attitude of Rāzī was therefore directly connected to his scientific views and was responsible for this transformation. In fact, his case marks one of the clearest examples of how philosophical and religious questions have played a role in many significant developments of science and in the history of science in general, displaying the intimate relation between man's view toward the sciences of nature and his vision of Reality as such.

Islamic civilization however rejected the philosophical views of Rāzī and his like and remained faithful to its own ethos and the burden which the hands of Providence had placed upon it, namely to bear the Divine Message of the Qur'ān for mankind to the end of the world. This truth has allowed Islam to preserve to this day, despite all the vicissitudes of time, the knowledge and practice of an inner alchemy which makes possible the cultivation of

Rāzī and his rejection of the alchemical view, see Corbin (with the collaboration of S. H. Nasr and O. Yahya), *Histoire de la philosophie islamique* (Paris, 1964) pp. 194-201 On the alchemy of Jābir see Corbin, "Le 'Livre du Glorieux' de Jābir ibn Ḥayyān (alchimie et archétypes)", *Eranos-Jahrbuch* (Zurich, 1950).

Throughout these works, there is a description and classification of mineral substances, chemical processes, apparatuses, and so forth, so that these works could be easily translated into modern chemical languages. There is no interest in the symbolic aspect of alchemy, in the discussion of metals and their transformations as symbols of the transformation of the soul. The correspondence between the natural and spiritual worlds which underlies the whole world-view of alchemy[15] has disappeared, and we are left with a science dealing with natural substances considered only in their external reality, albeit the language of alchemy and some of its ideas are still preserved.

The reason for Rāzī's departure from the alchemical view must be sought in the peculiar philosophical position which he held. As we know from many later sources including Bīrūnī, who was scientifically sympathetic with him. Rāzī wrote several works against prophetic religion and even denied prophecy as such.[16] He thus rejected a central theme of Islamic philosophy which in fact is "prophetic philosophy". Moreover, Rāzī was particularly opposed to Ismā-ʿīlism and carried out a series of highly philosophical debates with one of the leading figures of Ismāʿīlism, Abū Ḥātim Rāzī.[17] When the religious and philosophical attitudes implied by Rāzī's position are analyzed, it becomes clear why he transformed Jabirian alchemy into chemistry.

According to Islamic esotericism in general and Shīʿism – of which Ismā-ʿīlism is a branch - in particular, the sciences of nature are related to the science of revelation. Revelation possesses an exoteric (*ẓāhir*) and an esoteric (*bāṭin*) aspect and the process of spiritual realization implies beginning from the exoteric and reaching ultimately the esoteric. This process is called *ta'wīl* or hermeneutic interpretation, which is applied by the Shīʿah, and also in Sufism, to the Holy Quran, in order to discover its inner meaning. Only prophecy and revelation can enable man to make this journey from the exterior to the interior, to perform this *ta'wīl* which also means a personal transformation from the exterior man to the inner one.[18]

world view, there was no completely secularized domain of nature to which a totally "non-symbolic" science could apply. Therefore, although much chemistry was contained in the medieval alchemical tradition, especially in the case of Rāzī, it was never totally divorced from alchemy.

The *Sirr al-asrār* was translated and thoroughly studied by J. Ruska, *Al-Razi's Buch Geheimnis der Geheimnisse* (Berlin, 1937).

15. Concerning this correspondence see T. Burckhardt, *op.cit.*

16. One of Rāzī's famous works on this subject is the *Refutation of Prophecy, (al-Radd ʿala'l-nubuwwah)* See Bīrūnī, *Epitre de Beruni contenant le repertoire des ouvrages de Muhammad b. Zakariya al-Razi*, trans. et ed. P. Kraus, (Paris, 1936).

17 See P. Kraus, "Raziana", *Orientalia*, 4 (1935), 300-334; 5 (1936), 35-56, 358-370. The complete debate between the two Rāzī's, which centers mostly around the question of prophecy, runs throughout the many chapters of *Aʿlām al-nubuwwah (Peaks of Prophecy)*, ed. by S. al-Sawy and Gh. Aavani, (Tehran, 1977) Later Ismaʿīlī authors such as Ḥamīd al-Dīn Kirmanī in his *al-Aqwāl al-dhahabiyyah* and Nāṣir-i Khusraw in his *Jāmiʿ al-ḥikmatayn* were to continue this debate.

18. This theme has been thoroughly studied in the many writings of H. Corbin. As far as it concerns

And in fact, there is both similarity and difference when their alchemical and chemical ideas are compared.

Jābir believed that the elixir contained animal and plant substances as well as minerals, while Rāzī limited it to minerals and only casually mentioned animal and plant substances.[7] Rāzī divided metals into seven species including *khārṣīnī* just like Jābir in his *Kitāb al-khamsīn*. However, contrary to Jābir, Rāzī showed no interest in the numerical symbolism connected with this division. Jābir sought to discover the ultimate causes of things, while Rāzī, following the views of the Peripatetics among the physicians, denies openly that such a possibility exists.[8] Rāzī in his *al-Madkhal* and *al-Asrār* did not follow the Jābirian view that minerals are composed of sulphur and mercury but believed that they are constituted of body (*jasad*), spirit (*rūḥ*) and soul (*nafs*).[9] However, the Jābirian belief that there are five principles – the first substance, matter, form, time and space – certainly bears close resemblance to the famous five eternal principles of Rāzī.[10]

Rāzī also closely followed the terminology of Jābirian alchemy. He adopted not only technical names from Jābir but also titles of books. A large number of Rāzī's writings in this field bear the same titles as those of Jābir, while some are simply modifications of names of works belonging to the Jābirian corpus.[11] This is particularly significant in the case of such an independent philosopher as Rāzī. Even in the classification of simples (*ᶜaqāqīr*), which is among the most important scientific achievements of Rāzī in the field of chemistry, he followed the example of Jābir's *al-Usṭuqus al-uss al-awwal*.

One may then ask why Rāzī's works have been called the first books of chemistry in the history of science.[12] We have several extant alchemical works of Rāzī, such as *al-Madkhal al-taᶜlīmī* which served as a basis for the section on alchemy of *Mafātīḥ al-ᶜulūm*,[13] and most important of all, the *Sirr al-asrār*, well-known to the Western world as *Liber Secretorum Bubacaris*.[14]

7. Kraus, *op. cit.*, p. 3.

8. Kraus, *op. cit.*, p. 95, cites from Rāzī's *Kitāb al-khawāṣṣ* to this effect.

9. Stapleton, *op. cit.*, pp. 320 ff

10. Kraus, *op. cit.*, p. 137. Regarding the five eternal principles of Rāzī and his general philosophical views, see R. Walzer, *Greek into Arabic*, pp. 15-17.

11. Stapleton, *op. cit.*, pp. 336-337, where he cites fifteen works of Rāzī which have either identical or modified titles of works of Jābir and seem to deal with the same subject.

12. Stapleton, *op. cit.*, p. 320

13. The text of this work has been translated with commentary by Stapleton in the above-mentioned articles.

14. This work, whose title may have also been *Kitāb al-sirr* as cited by Ibn al-Nadīm, is the most basic work of Rāzī on chemistry, one in which the transformation of alchemy into chemistry may be clearly discerned. It was well-known during the later centuries in the Islamic world not only in its original Arabic version, but also in a Persian recension, and it was also influential in the West. But everywhere it was considered an alchemical work rather than a chemical one because, in the medieval

terials wed to the crafts and guilds.[3] Yet, it was also in Islam that the first seeds of a science of chemistry were sown, although the symbolic view of nature predominated and never allowed a secularized view of material substances to become dominant, for it is not possible to have a chemistry until the living body of nature has become converted into a cadaver and until nature has become deprived, for him who has lost the symbolist spirit, of the sacred presence which nevertheless continues to glow within all things.

The appearance of chemistry is related to the birth of a school of philosophy at the margin of Islamic intellectual life, and is bound to a change in intellectual perspective which corresponds directly to the profound difference between the world views of alchemy and chemistry. Moreover, the creation of this peripheral philosophical school and the birth of chemistry belong to the early period of Islamic history and concern two of the most famous figures of Islamic science, namely, Jābir ibn Ḥayyān, the Latin Geber (d. 3rd/9th century), and Muḥammad ibn Zakariyyā' Rāzī, the Latin Rhazes (d. 4th/10th century).

No two figures are better known in the annals of Islamic alchemy than these two men of many-sided genius. Both men were celebrated masters of alchemy. Both are believed to have belonged to the same school by later generations of alchemists in the Islamic and Western world.[4] Yet a study made of the writings of both men clearly reveals that although Rāzī employed the languages of Jābirian alchemy, he was in reality dealing not with alchemy but with chemistry. One might even say that Rāzī transformed alchemy into chemistry, even though alchemy endured long after him and chemistry continued to be cultivated in the Islamic world within the bosom of alchemy. Thus the chemistry of Rāzī was by no means independent of alchemy,[5] and in fact the two never parted ways completely in Islamic civilization as was to happen in the West after Robert Boyle.

Before discussing the philosophical and religious divergences between Jābir and Rāzī which led also to the separation of chemistry from alchemy, it is worthwhile to note the similarities and differences in the alchemical views of the two authors. Or rather, a comparison must be made between the Jābirian corpus, of which certainly much was written by Jābir himself and some of the treatises added later by Ismā'īlī authors, and the writings of Rāzī. Scholars studying these writings differ as to how closely Rāzī followed Jābirian alchemy[6]

3. See H. Corbin, *En Islam iranien*, vol. IV, (Paris, 1978), pp. 205 ff.

4. *Rutbat al-ḥakīm* considers Rāzī to be a disciple of the school of Jābir, while in almost all Latin alchemical texts the names of both men appear as unquestionable masters of alchemy.

5. See G. Heym, "Al-Rāzī and alchemy", *Ambix*, 1 (1938), 184-191; and J. R. Partington, "The Chemistry of Rāzī", *Ambix*, 1 (1938), 192-196.

6. For example, P. Kraus in his *Jābir ibn Ḥayyān*, vol. II, pp. 3 ff., does not believe that there is any direct and close relation between them, while N. F. Stapleton in "Chemistry in 'Iraq and Persia in the Tenth Century A.D.", written with R. F. Azo and M. Hidayat Husain, *Memoirs of the Asiatic Society of Bengal*, 1927, pp. 317-415, considers Rāzī as a direct disciple of Jābir.

Islamic Alchemy and the Birth of Chemistry

Seyyed Hossein Nasr*

ALCHEMY is at once a science of the cosmos, or cosmology, a sacred science of the soul, or psychology, a science of materials and a complement to certain branches of traditional medicine. It is not a proto-chemistry although it deals with physical materials from a particular point of view; nor is it the origin of the modern scientific method–although alchemy has been concerned in the profoundest sense with experiment and experience, that inner experiment which alone leads to certitude and of which all external experience is but a pale shadow.[1] The traditional alchemist serves as the window through which the light of the spiritual world shines upon the natural domain and the revivifying air or more precisely ether of the empyrean penetrates the arteries of nature. His aim is not to work with sheer material substances from a purely physical point of view, this being the work of charcoal burners. Rather, he aims to transform nature in order to return nature to that primordial perfection, that paradisal beatitude which nature is in reality, although this face of nature remains veiled and hidden from the view of modern man. Through the transmutation, based upon a sacred science of things, of the soul of the beholder to pure gold, alchemy permits the solar element or the supernal Apollo to shine upon the world of the gross elements and their compounds.

These general remarks on alchemy pertain as much to Islamic alchemy as to the Alexandrian or Latin schools, for all schools of traditional alchemy share ultimately the same world view and even the same symbolic language; although each of course possesses certain distinct characteristics. Islamic alchemy inherited at once Alexandrian and Chinese alchemy and created that immense synthesis. The translation of some of their fruits into Latin in the form of such texts as the *Turba Philosophorum* and *Picatrix*[2] brought Latin alchemy into being.

Islamic alchemy has managed to preserve over the centuries and even to our own day an integral spiritual alchemy wed to Sufism and other esoteric schools, such as that of the Shaykhīs in Persia, and a symbolic science of ma-

*The Iranian Academy of Philosophy, 6 Nezami Street, Avenue Francais, P. O. Box 14, 1699, Tehran, Iran.

1. On the alchemical tradition and its spiritual significance see T. Burckhardt, *Alchemy. Science of the Cosmos, Science of the Soul*, trans. W. Stoddart (Baltimore, 1971), and E. Zolla, *Le meraviglie della natura - Introduzione all'alchimia* (Milan, 1975).

2. On Islamic alchemy see S. H. Nasr, *Islamic Science - An Illustrated Study* (London, 1976), pp. 193 ff., and S. H. Nasr, *Science and Civilization in Islam* (New York, 1970), pp. 242 ff.

ment to a Jewish prayer book published in Venice in 1520 we learn that R. Abraham ben Yom Tov Yerushalmi used the tables of Ulugh Beg. It is otherwise known that this R. Abraham was in Istanbul in 1510.[33]

10. As a result of a comprehensive search of manuscript collections for Hebrew astronomical tables, some of the fruits of which have been presented here, it now appears that Levi ben Gerson (southern France, d. 1344) was the only Hebrew author to construct tables based on original models, rather than modifying or copying existing tables.[34] Moreover, his tables are embedded in a text that describes his models and their derivation from specified observations. In most other cases we find an introduction preceding the tables in which only the procedures for using them are indicated–this holds true for a large number of Islamic tables as well as those in Hebrew. Levi was certainly indebted to his Muslim predecessors, particularly al-Battānī whom he often cites as his source for tables representing Ptolemy's models. Levi also mentions al-Biṭrūjī but rejects his models categorically, preferring to take those of Ptolemy as his point of departure. In a general sense Levi's entire research program was an outgrowth of the Arabic scientific tradition, for his goal was to construct a system that was philosophically sound and mathematically rigorous. This view was expressed by a number of his predecessors including Ibn al-Haytham (Egypt, eleventh century), Ibn Bājja (Spain, twelfth century), Averroes (Spain, twelfth century), and al-Biṭrūjī. Carrying through with these ideas, Levi not only originated new planetary models, but proceeded to construct new tables, based on his models. Although Levi's astronomical treatise was translated into Latin, the extant manuscripts of that version contain few of the tables that belong to it.

Conclusion: We can see that the process of transmission is complex and that it is not always the result of a specific plan. Some translators, such as Moshe Ibn Tibbon, had clear goals to bring a certain literature to the attention of a recognizable group,[35] but in most cases we have too little information to make an informed judgment of the translator's motivation. What seems to emerge is a sense that in the late middle ages astronomy took on the character of an international enterprise despite the language barriers that separated its practitioners.

33. B. R. Goldstein, *The Astronomical Tables of Levi ben Gerson* (Hamden, Ct., 1974), pp. 75-76.

34. On Levi, see Goldstein (*op.cit.*, n. 33). In addition to the Hebrew manuscripts listed there (pp. 74 ff.), I have found a Geniza fragment of Levi's *Astronomy*, chapters 97 and 98 (corresponding to Paris Hb. 724, fol. 177a.24 to 178a.14 and including the marginal note on 178a) in Jewish Theological Seminary of America, Ms. ENA 2905, fol. 1.

35. On Moshe Ibn Tibbon, see D. Romano, "La transmission des sciences arabes par les juifs en Languedoc", in *Juifs et judaisme de Languedoc*, eds. M.-H. Vicaire and B. Blumenkranz (Toulouse, 1977), pp. 363-386. For biographical information on a fourteenth century translator, see L. V. Berman, "Samuel Ben Judah of Marseilles", in *Jewish Medieval and Renaissance Studies*, ed. A. Altmann (Cambridge, Mass., 1967), pp. 289-320.

al-Shāṭir or his models, they do yield information on other important aspects of late Islamic astronomy, and one may yet find references to Ibn al-Shāṭir and the Maragha School in Hebrew.* The main center for Islamic astronomy in the fifteenth century was the observatory in Samarqand in Central Asia established by the Mongol ruler Ulugh Beg, himself a noted astronomer.[28] The scientific legacy of Samarqand reached Istanbul, where the study of astronomy flourished in the sixteenth century, and there is now some evidence that this tradition also reached Italy. A Hebrew manuscript (Paris 1091) uniquely preserves an anonymous undated Hebrew translation, without the introduction, of Ulugh Beg's tables originally composed ca. 1440, and indeed the observatory at Samarqand is specifically mentioned in it (folio 70a): "Table for half-daylight for the latitude of Samarqand at the place of the observatory" *(ha-rasad)*. Although the planetary tables are taken from Ulugh Beg's work, the star catalogue in this manuscript is not the famous list that became known to western scholars in the seventeenth century,[29] but an older list presumably from a Hebrew source because its epoch is given in the text as "the beginning of the sixth millenium", i.e. 5000 A.M. *(anno mundi)*, which corresponds to 1240 A.D. Both Arabic and Hebrew names are displayed for each of the 50 stars together with their longitudes, latitudes, and magnitudes (folios 73a-74a). In an unpublished description of this manuscript on deposit at the Bibliothèque Nationale in Paris, M. Georges Vajda dates this copy by means of paleographic evidence to about 1500 A.D. Based on the watermark which is a simple anchor I am confident that the paper was produced in Venice between 1477 and 1508.[30] The pages are arranged in quires of 12 folios numbered in the upper left corner, e.g. on folio 13a we find 2:1 (in Hebrew alphabetic numerals) meaning quire 2, folio 1, on 14a we find just the numeral 2, and so on to 18a where we find the numeral 6; then on folio 25a we find 3·1. The keeper of Hebrew manuscripts at the Bibliothèque Nationale informed me that this arrangement is typical for Italian manuscripts of this period.[31] Italy, of course, was an important scientific center at the time and it is possible that knowledge of eastern Islamic astronomy was brought to the attention of Christian scholars by Jews. Ulugh Beg is mentioned in a few Hebrew texts deriving from Istanbul and I think it most likely that this translation was made there in the latter half of the fifteenth century. Steinschneider noted that Elia Bashyasi (d. Istanbul 1490) mentioned Ulugh Beg's tables in a work published in Istanbul in 1530/1,[32] and in a supple-

28. See Kennedy *(op.cit.*, n. 2), pp. 166 f. A. Sayili, *The Observatory in Islam* (Ankara, 1960), pp. 259-305.

29. See E. B. Knobel, *Ulugh Beg's Catalogue of Stars* (Washington, 1917), especially p. 9.

30. Cf. A. Mosliun, *Anchor Watermarks* (Amsterdam, 1973), especially plate 19, no. 233. Another text is bound with these tables to form Paris Ms. Hb. 1091, and its paper has a completely different watermark.

31. Cf. M. Beit Arié, *Hebrew Codicology* (Paris, 1976), p. 48.

32. Steinschneider *(op.cit.*, n. 1), p. 196.

* *Note added in proof.* In July 1979 I discovered a copy of Ibn al-Shāṭir's *zij al-jadīd* in Hebrew characters: JTSA Mic 2580 (cf. Ms. Oxford, Bodleian Arabic *Arch. Seld.* A 30). A note on the flyleaf in the same hand as the rest of the manuscript gives the solar, lunar, and planetary radices for 1260 AH (1844 AD) for Aleppo, and on internal evidence it seems to be a nineteenth century copy: in the mean motion tables entries are listed for 750, 900, 1050, 1200, 1230, 1260, 1290 AH (e.g. fol. 16b). This certainly suggests that the copyist (or his mentor) lived in the thirteenth century of the Hijra, i.e. the nineteenth century of the Christian era. It is surprising to find such a late copy of this text in Hebrew characters.

geographical coordinates are given as 72°E, 38°N.[22] Shelomo ben Eliyahu had the nickname "golden sceptre" *(sharvit ha-zahav)*, an allusion to *Esther* 4:11, and Steinschneider conjectured that there was an intention to find a biblical parallel to the Greek name Chrysococces;[23] this seems to be confirmed by the character of the text. In the introduction to the Hebrew version (Paris, Ms. Hb. 1042) we learn that the tables are arranged for the city Tivini (read: Tabriz) whose longitude is 72° rather than for Saloniki whose longitude is given as 49¾°. The mean motions are displayed for Persian years and months with radix 720 Yazdejird, i.e. 1350 A.D. The tables for the planetary equations are all derived from the *Almagest*, but in a form introduced by Islamic astronomers that Kennedy has called "displaced (Ar. *waḍ*ʿ*ī*) equation tables".[24] As in Ptolemy five functions are tabulated for each planet, but here some are displaced vertically to eliminate negative entries, some horizontally, and some both vertically and horizontally such that the resultant equations are in agreement with Ptolemy's values. For example, Jupiter's first correction (fol. 64b) which is due to the argument of longitude (or *centrum*) is tabulated at degree intervals where the entry for 0° is 4;27°, the maximum entry 11;15° corresponds to arguments 246° to 252°, and the minimum entry 0;45° corresponds to arguments 70° to 78°. These values derive from the *Almagest* XI, 11, columns 3 and 4 with horizontal shift of 18° and a vertical shift of 6°; e.g. Ptolemy's value for an argument of 18° is -1;33° and 6° - 1;33° = 4;27°, the entry for argument 0° in our table. Jupiter's second correction (fol. 65a) which is due to the corrected anomaly is given at degree intervals where the entry for 0° is 12°, and the maximum entry 23;3° corresponds to arguments 99° to 103°. All the entries are exactly 12° greater than the corresponding values in the *Almagest* XI,11, column 6. Kennedy[25] showed that these displacements must satisfy an algebraic relationship: the sum of the vertical displacements equals the horizontal displacement, in this case 6° + 12° = 18°. This technique was already in use in the ninth century by the Muslim astronomer Ḥabash al-Ḥāsib and continued with many variants throughout the middle ages.[26]

9. There has been considerable interest in the possibility that eastern Islamic scientific material reached Europe at the time of Copernicus because his models resemble quite closely those of Ibn al-Shāṭir (Syria, fourteenth century).[27] Although the Hebrew texts I have studied do not allude to Ibn

22. Pingree *(op.cit.*, n. 21) pp. 143-144.

23. Steinschneider *(op.cit.*, n. 1), p. 179.

24. E. S. Kennedy, "The Astronomical Tables of Ibn al-Aʿlam", *Journal for the History of Arabic Science* 1 (1977), 14.

25. Kennedy *(op.cit.*, n. 24), p. 15.

26. Kennedy *(op.cit.*, n. 24), pp. 16 f.; H. Salam and E. S. Kennedy, "Solar and Lunar Tables in Early Islamic Astronomy", *Journal of the American Oriental Society* 87 (1968), 492-497.

27. Cf. Imad Ghanem and E. S. Kennedy (eds.), *The Life and Work of Ibn al-Shāṭir* (Aleppo, 1976).

pended his own tables to this text, but they are unrelated to the Alfonsine Tables. The Hebrew translation of the Alfonsine Tables was not made until 1460 when Moshe ben Abraham de Nîmes translated them from Latin in Avignon together with the Latin introduction of John of Saxony (early fourteenth century), and so the Hebrew version is of no help in recovering the early history of the text.[16] There is another text in Hebrew, called the Paris Tables, based on the Alfonsine Tables and computed with radix 1368.[17] We read in this treatise that it was translated by Solomon ben Davin de Rodez in southern France (a pupil of Immanuel Bonfils of Tarascon), although no Latin title or author is cited. These tables are very extensive and make use of double arguments for finding the planetary longitudes and latitudes.[18] Some Latin texts are related to it: the earliest set of tables of this character are those of John of Lignières who worked in Paris about 1320. Although the principles underlying the computations are the same, all the entries differ because of a difference in convention. The entries in the planetary tables in this Hebrew text are, however, identical with those in an Oxford text by Batecombe (?) with radix 1348.[19] No copy of this Oxford text has been found in France, and no Latin version with radix 1368 and arranged for Paris, Lyons, and Avignon (as in the Hebrew version) is known.[20]

8. There were also translations of scientific works from the eastern Islamic world into Hebrew. Shelomo ben Eliyahu of Saloniki (fl. 1374-86) translated a text, called *The Persian Tables*, from Greek into Hebrew where the ultimate sources are the Sanjarī Zīj of al-Khāzinī (ca. 1120) and the ʿAlā'ī Zīj of al-Fahhād (ca. 1150).[21] The author of the Greek text, George Chrysococces, is not identified by Shelomo ben Eliyahu. In a passage written shortly after 1347, George Chrysococces tells us that he studied Persian astronomy with a Greek priest in Trebizond from whom he learned that a Greek scholar, Chioniades, had traveled to Persia to study astronomy and had brought back a number of texts which he then translated into Greek. Chrysococces wrote a commentary on these Persian tables of Chioniades which were constructed for Tabriz whose

16. On Moshe ben Abraham de Nîmes, see Steinschneider (*op.cit.*, n. 1), pp. 196 f.

17. I have consulted two copies of the Paris Tables: Munich, Hb. 343, fols. 104-167, and Oxford, Bodleian, Ms. Reggio 14, fols. 57-103. There is a Hebrew commentary on these tables by Moshe Farissol Botarel (southern France, ca. 1465), cf Oxford, Bodleian, Ms. Hb. 2022.

18. Cf. M. J. Tichenor, "Late Medieval Two-argument Tables for Planetary Longitudes", *Journal of Near Eastern Studies* 26 (1967), 126-128.

19. North (*op cit.*, n. 13), pp. 279 and 299 (n. 40). I have consulted two copies of the Latin version of these tables: Oxford, Bodleian, Ms. Rawlinson D.1227, fols. 64r-87r; and Bodleian, Ms. Laud Misc. 594, fols. 51r-81v.

20. Private communications from J. North, University of Groningen, and E. Poulle, École Nationale des Chartes, Paris.

21. On Shelomo ben Eliyahu, see Steinschneider (*op.cit.*, n. 1), pp. 178 ff. For the Greek version of the Persian Tables, see D. Pingree, "Gregory Chioniades and Palaeologan Astronomy", *Dumbarton Oaks Papers* 18 (1964), 135-160.

method of Maestro Campano for the meridian of Rome and Novara" (cf. TCD 49r: Tabula equationis lune). At the end of the Hebrew manuscript (folio 129b) one finds a page in Latin script but probably in Spanish; there is no heading and the few words are all technical terms: *Abril, Mayo, dias, altitud*, etc. It contains a somewhat confused version of a table of noon solar altitudes deriving from an Arabic or Hebrew original: in each entry the minutes precede the degrees indicating a thoughtless transcription from a script written from right to left. This table obviously was not taken from Campanus, and its source is unknown to me.

TABLE I

Paris Hb. 1102, 31a-32a		TCD, 63r
Mars [in Latin, Hebrew, and Arabic] Table for the mean motion of Mars in collected Christian years for the meridian of Novara in Italy		Tabula medii motus martis in annis domini iesu christi ad meridiem nouarie
Radix	2^s 17;46,15,0,0,0,0°	2^s 17;46,15°
28	1^s 7;6,34,49,5,29,27°	1^s 7; 6,35°
56	11^s 26;26,54,38,10,58,54°	11^s 26;26,55°
.		
.		
1512	1^s 12;4,5,10,56,30,18°	1^s 12;4,5°

7. The Alfonsine Tables were probably the most widely used tables in late medieval and renaissance Europe. The original form, based on Islamic models, was written in Spanish in the thirteenth century, but they were better known in the Latin version that appeared in the early fourteenth century.[13] Indeed the Spanish form does not seem to have survived. A Hebrew text by Isaac Israeli (ca. 1310), *Yesod Olam*, provides us with some background information: Isaac ben Sid, a Jewish astronomer who worked for King Alfonso of Castile, observed a solar eclipse in Toledo on 5 August 1263 to be about 7 digits in magnitude and he noted that the times of the eclipse phases were all a quarter-hour prior to the times predicted by the tables available to him (the Toledan Tables?).[14] He also observed three lunar eclipses at the request of King Alfonso: 24 December 1265, 19 June 1266, and 13 December 1266.[15] The discrepancies between observation and calculation were undoubtedly presented as part of the justification for constructing a new set of tables. Isaac Israeli ap-

13. J. North, "The Alfonsine Tables in England", in *Prismata: Festschrift für Willy Hartner*, eds. Y. Maeyama and W. G. Saltzer (Wiesbaden, 1977), p. 271.

14. Isaac Israeli, *Liber Yesod Olam*, eds. B. Goldberg and L. Rosenkranz (Berlin, part 1: 1848, part 2: 1846), part 2, 46b-47a.

15. Isaac Israeli (*op.cit.*, n. 14), part 2, 11b.

to al-Battānī in later Hebrew texts seem to derive from Bar Ḥiyya's adaptation rather than from a direct translation of the text. These tables were very popular in Hebrew, and they played much the same role as did the Toledan Tables for the Latin world–bringing technical astronomy to a new scientific community. It is puzzling that manuscripts of Bar Ḥiyya's Tables also contain tables ascribed to Abraham Ibn Ezra who lived somewhat later in the twelfth century. For example, one finds two tables of solar declination: one based on Ptolemy's value for the obliquity, 23;51,20°, ascribed to Abraham Bar Ḥiyya; and one based on the improved value, 23;33,8°, ascribed to Abraham Ibn Ezra. There are also many explanatory notes of a relatively trivial character written in the margins that are ascribed to Ibn Ezra as well. I have not found a separate set of tables in Hebrew composed by Ibn Ezra, though there are indications that they once existed.[9]

5. Al-Battānī's tables were also the basis for the popular tables, called *The Six Wings*, by Immanuel Bonfils of Tarascon (southern France, fourteenth century) who mentions his debt to his Muslim predecessor in the introduction.[10] These tables for computing conjunctions, oppositions, and solar and lunar eclipses use the Hebrew calendar with its nineteen-year cycle. Curiously, they were translated into both Latin and Byzantine Greek.[11] In this instance computations based on Ptolemy's models went from Greek into Arabic into Hebrew and then back into Greek.

6. Another set of tables related to those of al-Battānī can now be identified. A unique copy in Paris (Bibliothèque Nationale, Ms. Hb. 1102) contains an Arabic text in Hebrew characters that derives from the Latin text of Campanus of Novara (Italy) composed in the thirteenth century. This version in Hebrew script is anonymous and undated but seems to be from the fourteenth century. Its most important difference from the Latin version, at least the copy consulted by G. J. Toomer (Ms. TCD: Trinity College Dublin, D. 4.30), is that here the mean motions are expressed to six sexagesimal places whereas in the Latin they are only given to seconds (see Table I).[12] Campanus is mentioned in the Hebrew text (folio 93a): "Table for the equation of the moon according to the

9. Cf. J. M. Millás Vallicrosa (*op.cit.*, n. 8), p. 109 f.; and *idem*, *El libro de los fundamentos de las Tablas astronomicas de R. Abraham Ibn Ezra* (Madrid-Barcelona, 1947), pp. 59 ff.

10. The Hebrew text was published (Zhitomir, 1872), and a large number of manuscripts survive. Among the copies consulted in the course of this study is a fragment from the Cairo Geniza, Strasbourg Ms. 4045, fols. 20-22 (on fol. 22a the heading is faint but legible, "wing two").

11. On the Greek version of Bonfils' tables, see P. C. Solon, "The Six Wings of Immanuel Bonfils and Michael Chrysokokkes", *Centaurus* 15 (1970), 1-20. The only copy of the Latin translation is Florence, Biblioteca Nazionale, Ms. J IV 20 (the tables are on fols. 160r-182r). Despite the catalogue, Ms. Munich *cod. latin.* 15954 is a Hebrew copy of these tables in which the headings were translated into Latin.

12. I wish to thank G. J. Toomer, Brown University, for providing me with a detailed comparison of Ms. TCD with my notes on Paris Hb. 1102. The Latin manuscript is noted in F. S. Benjamin, Jr. and G. J. Toomer, *Campanus of Novara and Medieval Planetary Theory* (Madison, 1971), pp. 15-16.

star catalogue, and in most cases few of the figures were drawn. A partial exception is Paris Hb. 1019 (Anatoli's version) which has the chord table in Book I but otherwise, although lines are drawn for tables, no entries appear.[5] One wonders how working astronomers were able to make sense of the translation.

2. Southern France was the major center for translations from Arabic into Hebrew in the thirteenth and fourteenth centuries, and most of the texts that were in common use in Spain became available in Hebrew at that time. For example, Moshe Ben Tibbon, mentioned above, translated al-Biṭrūjī's *On the Principles of Astronomy*, written in Spain about 1200 A. D.[6] A Latin version by Michael Scot is also extant but it is much freer than the Hebrew. Al-Biṭrūjī attempted to harmonize Aristotelian cosmology with Ptolemaic astronomy by placing the geometric models for planetary motion on the surface of spheres rather than in the plane of the ecliptic. A number of later astronomers (some writing in Latin and others in Hebrew) found a variety of shortcomings in this synthesis and it was ultimately rejected. Several other scholars attempted to construct spherical models: for example, Joseph Ibn Nahmias (Spain, fourteenth century). His treatise was composed in Arabic (the unique surviving copy, in Hebrew characters, is Ms. V: Vatican Hb. 392), and translated into Hebrew anonymously (Ms. B: Oxford, Bodleian, Canon Misc. 334). His system was intended to be an improvement on that of al-Biṭrūjī, whom he cites (Ms. B 126v, 9, Ms. V 52b, 12), but the text awaits detailed analysis.

3. Another author whose work survives in Hebrew and Arabic versions is Joseph Ibn al-Wakkār (Spain, fourteenth century). He composed a set of astronomical tables for Toledo in Arabic and translated the introduction into Hebrew himself. In the unique surviving copy (Munich, Ms. Hb. 230) the Arabic text is in Hebrew characters and the Hebrew translation follows the Arabic. In the introduction Ibn al-Wakkār mentions the tables of Ibn al-Kammād which do not survive in the original Arabic, but only in a Latin version.[7] Ibn al-Wakkār's zīj is not mentioned in Kennedy's *Survey of Islamic Astronomical Tables* (1956).

4. The earliest set of astronomical tables in Hebrew are those of Abraham Bar Ḥiyya composed in Spain in the twelfth century.[8] His introduction is largely based on the introduction to al-Battānī's zīj (Syria, ninth century), and the tables agree very closely with those of al-Battānī as well. Indeed, the references

5. On folios 209b, 227b, etc., of Paris Hb. 1019 we find notes by Abraham ben Yom Tov Yerushalmi who lived in Istanbul in the sixteenth century (see paragraph 9, below).

6. See B. R. Goldstein, *Al-Biṭrūjī: on the Principles of Astronomy*, 2 vols. (New Haven, 1971).

7. J. M. Millás Vallicrosa, *Las Traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid, 1942), pp. 231 ff.

8. The introduction together with an excerpt from the tables was published by J. M. Millás Vallicrosa, *Libro del calculo de los movimientos de los astros de R. Abraham Bar Ḥiyya Ha-Bargeloni* (Barcelona, 1959).

new light on many aspects of Arabic science, and this will be illustrated here by concentrating on a few texts that I have studied in the past few years, several of which have been identified for the first time.

1. The translations from Arabic include works originally written in Greek such as Euclid's *Elements* and Ptolemy's *Almagest*. There are even two copies of the Arabic *Almagest* in Hebrew characters out of some ten extant copies: a complete copy with all the tables in a beautiful manuscript in Paris (Bibliothèque Nationale, Ms. Hb. 1100), and an incomplete copy in Cambridge (University Library, Ms. Mm 6.27 (8)).[3] Another manuscript (Vatican Hb. 392, folios 1-49) has been described as a copy of the Arabic *Almagest* in Hebrew characters, but in fact it is only a summary of it. The headings suggest that it is Ptolemy's work, for example we find "Book Four of the *Almagest*" (folio 5b), but later we find the heading "Book 7 and 8" (folio 28b), i.e. the entire discussion of the star catalogue is combined. Steinschneider[4] had queried whether this might be a copy of Ṭūsī's thirteenth century recension of the *Almagest*, but a comparison with British Museum Ms. Ar. Reg. 16 A VIII excludes that possibility, and the author of this text remains unidentified. There were two translations of the *Almagest* into Hebrew, one by Jacob Anatoli in Italy and the other by Moshe Ben Tibbon in southern France, both of whom lived in the thirteenth century. I have looked at quite a few copies of these translations and have been surprised to find that almost none of them has tables or the

including a few tables, by Yosef ben Yefet Halevi (fourteenth century) with a Hebrew translation, (2) a version of the zij of al-Fārisī (Yemen, thirteenth century) with tables, and (3) the *Tashīl al-Majisṭī* by Thābit Ibn Qurra. On the zij of al-Fārisī, see E. S. Kennedy, *A Survey of Islamic Astronomical Tables*, in *Transactions of the American Philosophical Society*, NS 46 (1956), p. 132. Arabic manuscripts of this zij are found in Cambridge (University Library, Ms. Gg 3.27) and Istanbul cf. M. Krause, "Stambuler Handschriften islamischer Mathematiker", in *Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik*, Abt. B, Vol. 3 (1936), p. 491. Two additional Arabic copies in Hebrew characters are preserved: Berlin, Ms. Hb. 682 Qu (cf. M. Steinschneider, "Schriften der Araber in hebräischen Handschriften", *Zeitschrift der deutschen morgenländischen Gesellschaft* 47 (1893), 355), and Jewish Theological Seminary of America, Ms. Hb. Micr. 2650 (the text is incomplete and only one table appears). F. Klein-Franke has given a brief description of a fragmentary Yemenite copy in Hebrew characters of al-Bīrūnī's *Elements of the Art of Astrology*, *Kiryat Sefer* 47 (1972), 720, in Hebrew). Cambridge University Library Ms. Add. 1191 contains two texts in Arabic written in Hebrew characters in a Yemenite hand. The first text is another copy of al-Kharaqī's *Kitāb al-tabṣira* (folios 1-18b); both the beginning and the end of this treatise are missing in this copy (cf. (b) above). The second text is Jābir ibn Aflaḥ's *Iṣlāḥ al-Majisṭī* (folios 19a-131a), and its colophon (f. 131a) gives the date of the copy as 1665 Seleucid Era (1354 A. D.), the beginning of this treatise is missing here. Another Arabic copy of this treatise in Hebrew characters is found in British Library Ms. heb. Or. 10,725 folios 92b-175b.

3. P. Kunitzsch lists nine copies of the Arabic Almagest in *Der Almagest: Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinischer Überlieferung* (Wiesbaden, 1974), pp. 34-46. The Cambridge manuscript, which is not mentioned there, follows the Isḥāq-Thābit version for the most part, but the Ḥajjāj version for Book VII 2-4 (cf. Kunitzsch, pp. 131 ff.). The star catalogue is missing and most of the tables come at the end, following Book XIII.

4. M. Steinschneider (*op. cit.*, n. 2), p. 359.

The Survival of Arabic Astronomy in Hebrew

Bernard R. Goldstein*

Introduction: Hebrew manuscripts are an important source for Arabic science, often containing texts that otherwise do not survive. Three types of texts can be distinguished: Arabic written in Hebrew characters, translations into Hebrew, and original Hebrew treatises based on Arabic models. In the areas where Arabic became predominant most Jews adopted it as their vernacular as well as their literary language. But beginning in the twelfth century, particularly in Spain, they began to use Hebrew for scientific and philosophical purposes. By the end of the middle ages we find such Hebrew texts being written in Spain, southern France, Sicily, Greece, and Turkey.[1] Moreover, we find Arabic texts in Hebrew characters from these places as well as from Egypt, Syria, and Yemen.[2] The study of this vast array of documents sheds

* The University of Pittsburgh (JS-2604 CL, Pittsburgh, PA 15260, U. S. A.) and The Institute for Advanced Study, Princeton.

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1. The best bibliographic study is still M. Steinschneider's *Mathematik bei den Juden* published in a series of articles between 1893 and 1901 and reprinted in a single volume (Hildesheim, 1964). See also E. Renan, "Les écrivains juifs français du XIVe siècle", in *Histoire Littéraire de la France*, Vol. 31, 1893.

2. (a) For Egypt we have a number of documents from the Cairo Geniza. See, for example, B. R. Goldstein and D. Pingree, "Horoscopes from the Cairo Geniza", *Journal of Near Eastern Studies* 36 (1977), 113-144.

(b) For Syria I have found only one astronomical text in Hebrew characters and it is from Aleppo, dated 1382 (Jewish Theological Seminary of America (*JTSA*), Ms. Hb. Micr. 2621, folios 1-23). The title is given in the colophon as *Kitāb al-tabṣira*. In fact, the text is *Kitāb al-tabṣira fī ʿilm al-hayʾa* by al-Kharaqī (d. 1138/39 in Merv) as I determined by comparing the manuscript in *JTSA* with a manuscript in the British Library (BL). The beginning of *JTSA* Ms. Hb. Micr. 2621, fol. 1a, corresponds to BL Ms. Add. 23394, fol. 99b:3 (Part 2, chapter 1, in the middle); the end of the *JTSA* ms. (fol. 23a) corresponds to the end of the BL ms. (fol. 110a: end of Part 2, chapter 14). The colophon of the *JTSA* ms. indicates that this copy was executed by David ben Joshua Maimuni, Nagid of the Egyptian Jewish community and a descendant of Maimonides, who left Egypt for Syria in the 1370s and is otherwise known to have been in Aleppo in 1375 and 1379 (*Encyclopedia Judaica* (1971), vol. 5, p. 1351). For a description of the Arabic text see E. Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte*, vol. 2, pp. 634 ff. (Hildesheim, 1970). On al-Kharaqī, see also *Encyclopedia of Islam*, 2nd ed., vol. 4, p. 1059.

(c) For Yemen, see Y. Ratzaby, "The Literature of the Yemenite Jews," *Kiryat Sefer* 28 (1952), 399-400 [in Hebrew]. Of special interest is British Museum Ms. Or. 4104, a Yemenite manuscript in Hebrew characters, which contains (1) an Arabic treatise on the motions of the sun and the moon,

the theorem of Ptolemy concerning a cyclic quadrilateral. He also used an expression for the area of an oblique triangle inscribed in a certain manner in a right triangle (cf. [7]). Abū al-Wafā' exhibits no knowledge of al-Shannī's work, although we have seen in the introduction that it is just possible that the former was required to use a method *different* from one already known.

Thus we have exhibited four algorisms for the area of a triangle, and five distinct proofs. Of course, by using algebraic techniques, it is not difficult to transform any one of the expressions into any other. But it must be remembered that similarities made obvious by algebraic symbols may not be apparent when the investigator is constrained to write out his rules in ordinary prose. This was the case with our ancient and medieval forebears.

Bibliography

1. Al-Bīrūnī, *Maqāla fī istikhrāj al-awtār fī'l-dā'ira*, the Leiden MS of this work was translated by H. Suter in *Bibl. Math.* 11(1910), 11-78; the Bankipore version has been published in a volume entitled *Rasā'ilu'l-Bīrūnī* (Hyderabad-Dn.: Oriental Publications Bureau, 1948), cf. the review of H. Hermelink, *Zentr. f. Math.*, 54 (1956), 3.
2. Clagett, Marshall, *Archimedes in the Middle Ages*, vol. 1 (Madison, Wis., 1964).
3. *Dictionary of Scientific Biography* (New York: Charles Scribner's Sons, 1970-1978).
4. Heath, T. L., *A History of Greek Mathematics*, 2 vols. (Oxford, 1921).
5. Heath, T. L., *The Thirteen Books of Euclid's Elements*, 2d. ed., 3 vols. (New York: Dover Publications, 1956).
6. Hermelink H., and E. S. Kennedy, "Transcription of Arabic Letters in Geometric Figures", *Journal of the American Oriental Society*, 82 (1962), 204.
7. Id, Yusuf, and E. S. Kennedy, "A Medieval Proof of Heron's Theorem", *The Mathematics Teacher*, 42 (1969), 585-7.
8. Al-Kāshī, Jamshid Ghyāth al-Dīn, *Miftaḥ al-Ḥisāb* (Tehran lithograph edition, 1889).
9. Luckey, Paul, "Zur Entstehung der Kugeldreiecksrechnung", *Deutsche Mathematik*, 5 (1940-1), 405-446.
10. Sezgin, Fuat, *Geschichte des arabischen Schrifttums*, Bd. V (Leiden: E. J. Brill, 1974).
11. Suter, Heinrich, "Die Mathematiker und Astronomen der Araber und ihre Werke", *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, X. Heft, Leipzig, 1900.

Combining (29) and (30),

$$\overline{AB}^2 \cdot \overline{GB}^2 - \left(\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2}\right)^2 = 2 \text{ area } \triangle ABC,$$

which is equivalent to (27a), hence (27).

Q.E.D.

The treatise closes with a curious passage (82v·36-38) in which the author remarks apologetically that areas should not be multiplied together, but that he has done so for the sake of simplication. His qualms are a vestigial remnant of the ancient geometrical algebra in which terms of the first degree represented line segments, quadratic terms areas, and cubic terms volumes. In his rules indeed many quartic elements appear.

The Background of the Problem

The earliest of the rules for calculating the area of a triangle in terms of its sides is the elegant

(31) $$\sqrt{s(s-a)(s-b)(s-c)}\ ,$$

where s is the semiperimeter. Although it is known as "Heron's Formula", its discovery is by Bīrūnī (in [1], transl., p. 39) attributed to Archimedes (c. 250 B.C.). However, Heron's "Metrica" (written c. 75 A.D.) contains a proof which employs the properties of the incircle, similar triangles, inscribed angles, and the properties of proportions ([4], vol. 2, pp. 34-35).

The same book proves a different rule, namely

(32) $$\frac{c}{2}\sqrt{a^2 - \left(\frac{c^2 + a^2 - b^2}{2c}\right)^2}\ .$$

This expression differs only slightly from Abū al-Wafā's third rule, (27) and Heron's proof is also strikingly similar, employing the same proposition from Euclid as does Abū al-Wafā'. Nevertheless, the latter does not mention Heron or anyone else in this connection.

Two proofs of formula (31) have been noted in the Arabic literature anterior to Abū al-Wafā'. The earlier (c. 875) is by the Banū Mūsā, and exhibits only trivial divergences from that of the Heronic Metrica ([2], pp. 279-289).

The second is by a certain geometer named Abū ʿAbdallah Muḥammad b. Aḥmad al-Shannī (c. 950). He uses the excircle, similar triangles, and a property of a broken line inscribed in a circle ([1], pp. 39-40). It is considerably more involved than Heron's proof.

In a different source the same al-Shannī states and proves expression (1), Abū al-Wafā's first rule. The two proofs differ widely, for al-Shannī applies

Two More Rules

After completing the proof, the text states that it is possible to calculate the area of a triangle by operations performed upon its sides as such. Expressed in modern symbols, the rule is

$$(26)\qquad \frac{1}{4}\sqrt{\{(c+b)^2-a^2\}\{(c-b)^2-a^2\}}\,. \qquad (82v:21\text{-}25)$$

No proof is given; perhaps it was felt that (1a) and (26) are sufficiently similar that proof of one suffices for the other.

The author goes on to say that there is yet another rule for the area of a triangle in which no altitude is employed; it is

$$(27)\qquad \frac{1}{2}\sqrt{c^2a^2-\left(\frac{c^2+a^2-b^2}{2}\right)^2} \qquad (82v:26\text{-}28)$$

For this a proof is given. Before presenting it we restate the expression above in terms of the capital letters on the figure. The text has a separate figure but the previous one will serve.

To prove

$$(27a)\qquad \frac{1}{2}\sqrt{\overline{AB}^2\cdot\overline{GB}^2-\left(\frac{\overline{AB}^2+\overline{GB}^2-\overline{AG}^2}{2}\right)^2}=\text{area } \Delta ABG$$

Proof:

$$\overline{AB}^2+\overline{BG}^2=\overline{AG}^2+2\cdot GB\cdot BD. \qquad (82v:29)$$

This is Proposition 13 in Book 2 of Euclid's Elements ([5], vol. 1, p. 406).

Hence

$$(28)\qquad BG\cdot BD=\frac{\overline{AB}^2+\overline{BG}^2-\overline{AG}^2}{2}\,. \qquad (82v:31)$$

Square both sides of (28) and subtract each side of the result from $\overline{AB}^2\cdot\overline{GB}^2$ to obtain

$$(29)\qquad \overline{AB}^2\cdot\overline{GB}^2-\overline{BG}^2\cdot\overline{BD}^2=\overline{AB}^2\cdot\overline{GB}^2-\left(\frac{\overline{AB}^2+\overline{BG}^2-\overline{AG}^2}{2}\right)^2.$$

The left hand side of (29) is

$$\begin{aligned}(\overline{AB}^2-\overline{BD}^2)\,\overline{BG}^2&=\overline{AD}^2\cdot\overline{BG}^2\\&=(\overline{AD}\cdot\overline{BG})^2 &&(82v:35)\\(30)\qquad&=(2\text{ area }\Delta ABG)^2,\end{aligned}$$

by application of the Pythagorean theorem to ΔADB, and the fact that AD is an altitude of ΔABG.

proportional, and the angle they enclose is common, the triangles are similar).

So angle BKY is a right angle (for triangle ZBT, similar to it, is inscribed in a semicircle).

Also $$TZ/K[Y] = TB/BK \quad .$$

(The text has KB. The segments are corresponding sides of similar triangles. Squaring both sides),

$$\overline{TZ}^2/\overline{YK}^2 = \overline{TB}^2/\overline{BK}^2. \qquad (82r:34)$$

Further,

(21) $$(\overline{TZ}^2 - \overline{KY}^2) / \overline{TZ}^2 \quad (\overline{TB}^2 - \overline{KB}^2) / \overline{TB}^2$$

(since if $x/y = u/v$, then $(x-y)/x = (u-v)/u$.)

But

(8) $$\overline{TZ}^2 - \overline{KY}^2 = \overline{AD}^2. \qquad (82r:35, 82v:1)$$

(This is the second lemma).

Moreover,

(22) $$\overline{TB}^2 - \overline{BK}^2 = \overline{[B]L}^2. \qquad (82v:1)$$

(The text has NL. To verify this, apply the Pythagorean theorem to triangle EBL to obtain $\overline{EB}^2 \cdot \overline{EL}^2 = \overline{BL}^2$, and to this apply (2), (3), and (5).)

So (applying (8), (22), and (2) to (21))

$$\overline{AD}^2/\overline{TZ}^2 = \overline{BL}^2/\overline{BE}^2. \qquad (82v:2)$$

And (since AD is the altitude to side a and $BE = a/2$)

(23) $$\overline{AD}^2 \cdot \overline{BE}^2 = \text{area } ABG^2 = \overline{TZ}^2 \cdot \overline{BL}^2.$$

Now

(24) $$\overline{TZ}^2 = \overline{BZ}^2 - \overline{BE}^2. \qquad (82v:3)$$

(This follows by combining with (2) the Pythagorean expression

$$\overline{TZ}^2 = \overline{BZ}^2 - \overline{BT}^2.)$$

Also

(25) $$\overline{BL}^2 = \overline{BE}^2 - \overline{AZ}^2,$$

(which is obtained by combining with (3) the Pythagorean expression $\overline{BL}^2 = \overline{BE}^2 - \overline{EL}^2$).

(Substitution of (24) and (25) in (23) gives

(1a) $$\text{area } \overline{ABG}^2 = (\overline{BZ}^2 - \overline{BE}^2)(\overline{BE}^2 - \overline{AZ}^2). \qquad (82v:4)$$

Q.E.D.

Now

(17) $$\overline{DE}^2 - \overline{AZ}^2 = \overline{KY}^2.$$

(To demonstrate this, use (4) and (5) to write $\overline{DE}^2 - \overline{AZ}^2 = \overline{BY}^2 - \overline{BK}^2 = \overline{YK}^2$, the last by applying the Pythagorean theorem to triangle YKB. It is proved to be a right triangle at 82r:33 without invoking the second lemma, so the demonstration is not circular).

Also

(18) $$\overline{BZ}^2 - [\overline{BE}^2 = \overline{TZ}^2].$$ (82v:21)

(Here use (2) to put

$$\overline{BZ}^2 - \overline{BE}^2 = \overline{BZ}^2 - \overline{BT}^2 = \overline{TZ}^2,$$

the last by applying the Pythagorean theorem to triangle ZTB).

Finally, application of (17) and (18) to (16) yields

(8) $$[\overline{TZ}^2 -]\ \overline{K[Y]}^2 = \overline{AD}^2.$$ (82v:21)

(The text has KG. A copyist apparently left out the few words so indicated from line 21, but the intent of the author is clear).

Q.E.D.

The Main Demonstration

(19) $$\overline{BZ}^2 - \overline{BE}^2 = \overline{TZ}^2$$ (82r:30)

(By the Pythagorean theorem, $\overline{BZ}^2 - \overline{TB}^2 = \overline{TZ}^2$, and invocation of (5) yields (19).)

$$\overline{BE}^2 - \overline{AZ}^2 = \overline{BL}^2.$$ (82r:31)

This follows from the Pythagorean expression $\overline{BE}^2 \quad \overline{EL}^2 = \overline{BL}^2$ and use of (3).)

The first lemma says

(6) $$HB/BG = DE/AZ.$$ (82r:31)

Hence

$$ZB/BE = DE/A[Z].$$ (82r:32)

(The text has AB. The expression follows from the fact that $HB = 2ZB$ and $BG = 2BE$). And (by use of (2), (4), and (5))

(20) $$ZB/BT = YB/BK.$$ (82r:32)

Hence

$$YK \parallel TZ$$ (82r:33)

(since by (20) two pairs of corresponding sides of triangles ZBT and YBK are

The Second Lemma

To prove:

(8) $$\overline{TZ}^2 - \overline{YK}^2 = \overline{AD}^2$$ (82v:15)

Proof:

(9) $$\overline{BZ}^2 + \overline{ZA}^2 = 2(BZ\cdot ZA) + \overline{AB}^2$$ (82v:15)

(This is immediate upon squaring the identity $BZ + ZA = AB$).

(10) $$2(B[Z]\cdot AZ) = BH\cdot AZ = BG\cdot DE.$$ (82v:16)

(The text has BE. The first equality is a consequence of the fact that $BZ = BH/2$. The second equality is equivalent to Lemma 1).

(11) $$\overline{AB}^2 = \overline{BD}^2 + \overline{DA}^2$$

(by application of the Pythagorean theorem to the right triangle ABD).

(12) $$\overline{BZ}^2 + \overline{ZA}^2 = 2(BE\cdot ED) + \overline{BD}^2 + \overline{DA}^2.$$ (82v:17)

(In the MS the first three terms are repeated. To obtain (12), note that by (10)

$$2(BZ\cdot AZ) = BG\cdot DE = a\cdot ED = 2BE\cdot ED,$$

and apply it and (11) to (5).)

But

(13) $$2(BE\cdot ED) = 2(B[D]\cdot ED) + 2\overline{DE}^2$$ (82v:18)

(The text has BE. Multiply both sides of the identity $BE = BD + DE$ by $2\ ED$ to obtain (13).)

Also

(14) $$\overline{BE}^2 = \overline{BD}^2 + \overline{DE}^2 + 2(BD\cdot DE)$$ (82v:19)

(This may be obtained by squaring both sides of the identity above, $BE = BD + DE$).

So

(15) $$\overline{BZ}^2 + \overline{ZA}^2 = \overline{AD}^2 + \overline{BE}^2 + \overline{ED}^2$$

(obtainable by taking (12) and eliminating from it $2(BE\cdot ED)$ by the use of (13). There results $\overline{BZ}^2 + \overline{ZA}^2 = 2(BD\cdot ED) + 2\overline{DE}^2 + \overline{BD}^2 + \overline{DA}^2.$ From the right hand side of this expression, pick the elements of the right hand side of (14), and substitute for them $\overline{BE}^2$, the left hand side of (14). There results (15).)

Or

(16) $$\overline{BZ}^2 - \overline{BE}^2 = \overline{AD}^2 + \overline{DE}^2 - \overline{AZ}^2$$ (82v:20)

to the relation between sides b and c. We have taken $b > c$, implying that both sides of expression (7) below are negative, a concept foreign to medieval mathematics. However, (7) is slightly misleading, for the Arabic word *faḍl* does not translate precisely as "difference", but rather as "the excess (of one quantity over another)". The proof is valid under all circumstances.

Construction

For the proof the text prescribes (82r:28) the dropping of altitude AD to a, and the drawing of semicircles BTZ and BLE with bounding diameters BZ and BE respectively.

Next the laying out of four line segments is called for (82r:29), all chords or portions of chords in the semicircles just drawn. They are:

(2) $BT = BE$

(3) $EL = AZ$

(4) $BY = DE$

(5) $[B]K = AZ$ (The text has YK).

The First Lemma

To prove:

(6) $$HB / BG = DE / AZ.$$ (82v:9)

Proof:

(7) $$\overline{BA}^2 - \overline{AG}^2 = \overline{BD}^2 - \overline{DG}^2,$$ (82v:10)

since, (by the Pythagorean theorem)

$$\overline{BA}^2 - \overline{BD}^2 = \overline{AD}^2 = \overline{AG}^2 - \overline{GD}^2.$$

(The above expression is evidently intended, but the passage is garbled and not easily restorable).

The right hand side of (7) is

$$\overline{BD}^2 - \overline{DG}^2 = (B[D] + [D]G)\,(B[D] - [D]G) = BG \cdot 2DE.$$

(The text has at 82v:12 $(BE + EG)(BE - EG)$, which is absurd).

The left hand side of (7) is

$$\overline{BA}^2 - \overline{A[G]}^2 = (BA + A[G])\,(BA - A[G]) = (c+b)(c-b) = HB \cdot 2AZ.$$ (82v:13)

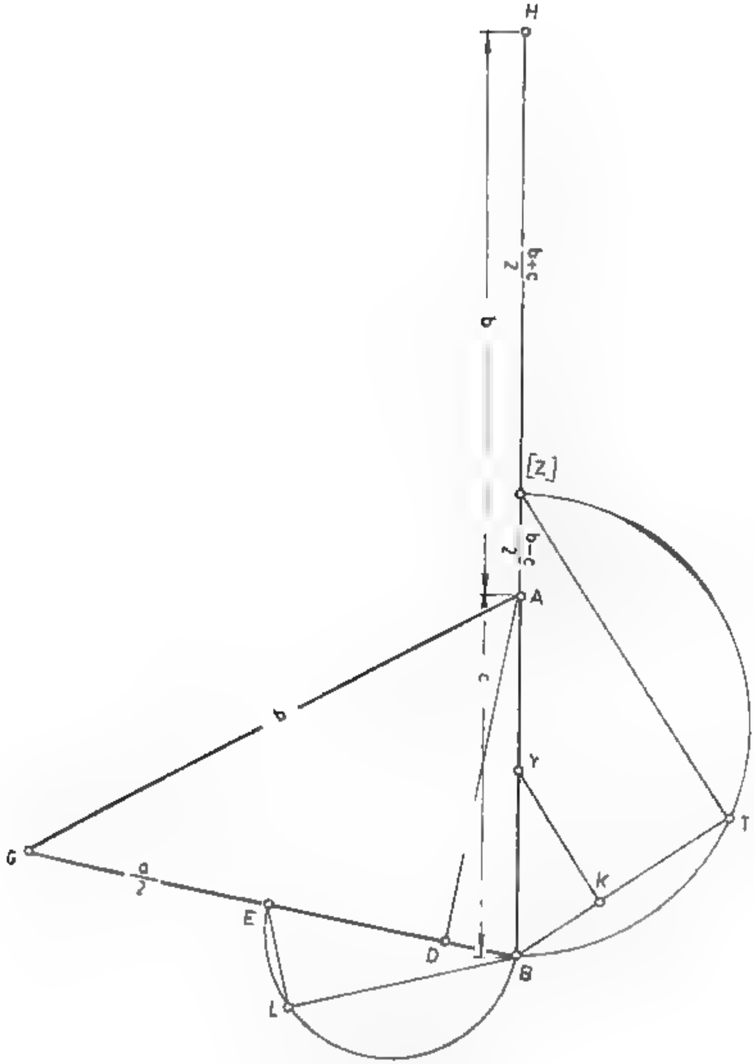
Hence

$$BG \cdot 2DE = 2AZ \cdot HB,$$

whence

(6) $$HB / BG = DE / AZ.$$ (82v:14)

Q.E.D.



Restored version of the text figure.

missing, and that the original version was a challenge to produce a proof different from one already current. Be that as it may, the verbal rule which follows is clear. Expressed in modern symbols it is

$$\sqrt{\left\{\left(\frac{c+b}{2}\right)^2-\left(\frac{a}{2}\right)^2\right\}\left\{\left(\frac{a}{2}\right)^2-\left(\frac{c-b}{2}\right)^2\right\}} \quad , \tag{1}$$ (82r:21)

where a, b, and c are the lengths of the sides of an arbitrary triangle. Passages in the text will be identified, as is the expression above, by a pair of numbers separated by a colon, the first giving the number and side of the folio, the second the line.

For the demonstration which follows, a figure is utilized, transcribed on page 23 below. The Arabic letters of the MS have been replaced on our figure by Latin characters according to the system given in [6].

To prove (1) Abū al-Wafā' makes additions to the figure and then, with the aid of two lemmas, goes through a long series of deductions which eventually yield what is desired. The next three sections below duplicate his argument, except that we have compressed his verbal statements into symbolic expressions, and whereas he leaves the proofs of the lemmas until after the main theorem has been disposed of, we prove the lemmas first.

The text has two more rules giving the area of a triangle in terms of its sides, there being a proof for the second rule of the two. This material also is paraphrased by us below.

But, of course, the problem of determining the area of a triangle in terms of its sides is far older than Abū al-Wafā'. It apparently reaches back to Archimedes, and between his time and the tenth century several rules and variant proofs were worked out. The concluding sections of our paper list these rules in approximate chronological order and discuss the relations between them.

Enunciation of the Theorem

In the triangle ABG, (82r:25, see our version of the figure) extend AB to H, making $AH = AG = b$. Bisect BH at Z and BG at E. It is to be proved that

$$(\overline{BZ}^2 - \overline{BE}^2)(\overline{BE}^2 - \overline{AZ}^2) = \overline{\text{area } ABG}^2 \;. \tag{1a}$$ (82r:27)

Since from the figure $BZ = (c+b)/2$, and

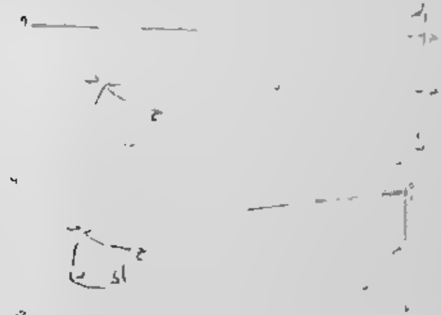
$$AZ = BZ - AB = \{(b+c)/2\} - c = (b-c)/2,$$

expressions (1) and (1a) are equivalent.

In the text figure, which has apparently suffered at the hands of successive copyists and is grossly inaccurate, AB has not been extended, so no H appears. Where Z should be, a second D has been written (the cognate Arabic letters *za'* and *dāl* resemble each other). There is no indication in the text as

Ẓāhiriya MS 4871, f. 82v.

[illegible]



[illegible]

Ẓāhiriya MS 4871, f. 82r

Abū al-Wafā' and the Heron Theorems

E. S. KENNEDY* AND MUSTAFA MAWALDI*

Introduction

MANUSCRIPT 4871 of the Ẓāhiriya Library in Damascus contains a number of Arabic translations of philosophical tracts from late antiquity. Several of these have been published. What is less well known is that the same manuscript includes many scientific works, in great part unique, and of considerable historical interest.

This paper discusses the contents of one of them, a short treatise which covers most of a single folio only, 82, reproduced in facsimile here on pages 20 and 21 by kind permission of the librarian of the Ẓāhiriya.

Two individuals are mentioned at the beginning of the treatise, both being known to historians of the exact sciences. The first, the presumed author of the writing, is the famous Abū al-Wafā' al-Būzjānī (940-998), a mathematician and astronomer of Khurasanian origin who lived and worked in Baghdad ([3], vol. 1, pp. 39-43. Here and in the sequel, references enclosed in square brackets are to the numbered bibliography at the end of the paper. However, any square brackets which appear in algebraic expressions denote restorations of errors or omissions in the Arabic text of the MS).

The second is one Abū ʿAlī al-Ḥasan b. Ḥārith al-Ḥubūbī, here called a canon lawyer *(faqīh)*, in other contexts given the title of judge ([11], p. 197; [10], p. 336). He was evidently a contemporary of Abū al-Wafā', as our text bears witness. Beyond this, Abū Naṣr Manṣūr b. ʿIrāq (in [9], p. 424) mentions a letter sent by Abū al-Wafā' to al-Ḥubūbī concerning some developments in spherical trigonometry. Al-Bīrūnī in his treatise on chords ([1], transl., p. 17), gives two proofs by al-Ḥubūbī of a certain theorem. Al-Kāshī ([8], p. 229) attributes to him a method of solving problems in the algebra of inheritances. Al-Kāshī calls him al-Khwārizmī, thus implying that he or his antecedents stemmed from the region south of the Aral Sea.

The Ẓāhiriya MS states that al-Ḥubūbī requested from Abū al-Wafā' a proof of the rule for calculating the area of a triangle without having recourse to an altitude. Here the text seems to be corrupt. It is possible that a clause is

*The American Research Center in Egypt, 2 Maydan Qasr al-Dubara, Cairo, Egypt, and the Institute for the History of Arabic Science, Aleppo University, Aleppo, Syria. That part of the study carried out at the ARCE was supported mainly by the Smithsonian Institution. The authors also acknowledge with gratitude assistance given them by Professors Adel Anbouba and M.-Th. Debarnot, who rescued them from an egregious blunder in restoring the text figure.

فقد تبيّن ما قلنا انه متى تحركت نقطة ه بمجموع الحركتين المذكورتين حصل لها حركة مستوية بالنسبة الى نقطة د ومساوية في السرعة لحركة دائرة ل‌نم .

فاذا فرض البصر على نقطة ق من خط طج وفرض بعده من ط مساويا[19] لبعد نقطة ط من نقطة د فإن هذه الابعاد متى كانت مقاديرها على وفق الاقدار التي وضعها

[١٦٠ و]

بطلميوس لبعدي مركزي الحامل والمعدّل من نقطة ق اعني مركز العالم في واحد من الكواكب كان ما يظهر من هذه الحركات موافقا لما يظهر له بالارصاد .

١٩ – مساو

ولتكن هذه الكرة مغرقة في ثخن كرة تحدها سطحان متوازيان مركزهما نقطة ك ، فتماس[17] سطحيها المتوازيين بحيث يماس سطح المدير سطحيها الظاهر والباطن . وتسمى هذه الكرة الفلك الحامل .

فاذا تحركت هذه الكرة دورة تامة رسم مركز المدير دائرة مركزها نقطة ك وهي الدائرة الوسطى المذكورة .

واذا تحرك المدير على مركز ب رسم تدوير الكوكب اعني نقطة ه الدائرة الصغيرة التي في داخل كرة المدير اعني دائرة اس‍ه المذكورة .

واذا تحرك الحامل تحركت نقطة ل محيط دائرة لدم الثالثة التي مركزها نقطة ك حركة مستوية فانها تدير[18] بدورانها كرة المدير . فيدور بدوران كرة المدير مركز التدوير على دائرة اس‍ه الصغيرة على مركزها اعني نقطة ل حركة مستوية ايضا ومساوية في السرعة لحركة نقطة ل .

فاذا انتقلت نقطة ل على دائرة لدم الى د ثم الى م في النصف الايسر من دائرة لدم انتقلت نقطة ه على دائرة اس‍ه في النصف الايمن من دائرة هسا الى نقطة ع ثم الى نقطة ح .

واذا تصورت هذا الامر على ما شرحناه فإنّ مركز المدير ومركز التدوير عن اي وضع فرضناه . ووصلنا خطوط لكفن ، درع ص ، نع ت الى محيط التدوير .

فاقول إنّ خطّي لكفن ، درع ص متوازيان .

برهانه انّ قوس لد من دائرة لدم تكون في جميع اوضاع نقطة ل اعني ن من دائرة لدم شبيهة بقوس هع من الدائرة الصغيرة . فزاويتا ملكن ، فنع متساويتان . فخطا كن دع متوازيان . فزاوية ادع مثل زاوية للكن . فحركة نقطة ه اعني ع على مركز د شبيهة بحركة نقطة ل اعني ن على مركز ك في اي وضع وزمان فرض .

لكن حركة نقطة ن على مركز ك حركة مستوية فحركة نقطة ع على مركز د عني مركز معدل المسير حركة مستوية . وهذه الحركة التي حصلت لنقطة ع على مركز د حركة مركبة من حركتي نقطتي ل ه اعني ن ع المستويتين .

١٧ — يماس .
١٨ — يدير .

مركزها اقرب من النقطة التي عليها البصر من أجل أنّ مركز التدوير يكون على هذه الدائرة في بعديه المختلفين اعني اعظم ابعاده من البصر واقربها منه .

وكونها قريبا من محيطها في باقي ذروته حداً فلذلك ظنّ بطلميوس أنّ مركز التدوير لازماً لمحيطها وانه يرسمها بحركته .

[12]ولنضرب لذلك مثالاً ليظهر ظهورا بينا . فليكن دائرتان متساويتان في بسيط واحد متقاطعتان . الاولى منهما وهي يجعلها بطلميوس دائرة معدّل المسير عليها ابجد مركزها[13] نقطة د . والثانية منهما وهي التي يجعلها الفلك الحامل لمركز التدوير دائرة هزح ومركزها نقطة ط . وليتقاطعا على نقطتي وي . ونصل خط دط المار بالمركزين وننفذه في الجهتين الى محيطها . وليقطع دائرة ابجد على نقطتي اج ودائرة هزح على هح . ونقسم خط دط بنصفين على نقطة ك ونجعلها مركزاً وندير عليها دائرة وببعد دا اعني نصف قطر الدائرة الاولى عليها لنم . فتقطع كل واحد من خطي اه ، حج بنصفين على نقطة لم.

فنجعل نقطة ل مركزاً ونديرببعد ال دائرة صغيرة عليها اسره . فتماس[14] دائرة ابجد من داخل على نقطة ا وتماس دائرة هزح من خارج على نقطة ه . ولتكن[15]

[١٥٩ ظ]

نقطة س في النصف الايمن من الدائرة الصغيرة .

فمن البيّن أنّ نصف قطر هذه الدائرة اعني ال يكون مساويا لخط دك اعني نصف الخط الذي بين مركزي دائرتي ابجد ، هزح الاولتين

فاذا توهمنا أنّ دائرتي ابجد ، هزح الاولتين ثابتتين وان الكرة المحيطة بتدوير الكوكب يماس[16] سطحها سطح التدوير يكون مركزها نقطة ل وتسمى هذه الكرة الفلك المدير للتدوير .

١٢ – النص : من هب وصعداً : هو عينه النص الذي ورد في نهاية الادراك لقطب الدين الشيرازي مع تغيرات طفيفة جداً لم تؤثر على الهيئة التي توهمها .

١٣ – مكرورة .   ١٤ – فيماس .

١٥ – وليكن . يلي هذه الكلمة شكل يخال انه يمثل هذه الدوائر غير انه مرموز اليه على هامش الصفحة بالعبارة التالية : " هذا الشكل خطأ " . لذلك اعدنا رسمه حسب مقتضيات النص انظر الشكل ٢ الملحق بهذا المقال .

١٦ – تماس .

وتقطع هذه الدائرة كل واحدة[9] من الدائرتين الاولتين على نقطتين غير نقطتي تقاطع الدائرتين الاولتين .

فاذا جعلنا موضع قطع هذه الدائرة لاحد قسمي الخط الذي فيما بين الدائرتين مركزاً وادرنا عليه دائرة صغيرة تماسّ الدائرتين الاولتين ، فإن قطر هذه الدائرة يكون مساويا لبعد ما بين مركزي الدائرتين الاولتين .

فمتى تحرك مركز هذه الدائرة الصغيرة على محيط الدائرة الثالثة وهي الوسطى من الدوائر الثلثة المتساوية الى ان يصير وضعها على هذا الخط من الجهة الاخرى مقاطراً لهذا الوضع فإن الدائرة الصغيرة تصير ايضاً مماسّة للدائرتين اللتين كانت مماسّة لهما في الوضع الاول من داخل ومن خارج فتماس التي كانت تماسها من داخل من خارج وبالعكس في الاخرى .

واذا توهم مركز فلك تدوير الكوكب محمولاً على محيط هذه الدائرة الصغيرة وفرضت متحركة على مركزها امّا في القوس العليا منها فالى التوالي اعني الجهة التي يتحرك مركزها اليها ، واما في القوس السفلى بالعكس وفرضت الحركتان[10] متساويتين[10] وفرضت الدائرتان[11] الاولتان[11] ثابتتين وفرض البصر على الخط المار بالمراكز وبعده من مركز احدى الدائرتين الاولتين مثل بعد ما بين مركزيهما . فاذا توهم مركز تدوير الكوكب على النقطة التي تماس الدائرة الصغيرة احدى الدائرتين الاولتين من خارج اعني التي مركزها اقرب من النقطة التي توضع عليها ثم تحركت الدائرة الصغيرة فحركت بحركتها النقطة المماسة اعني مركز التدوير الى خلاف الجهة التي يتحرك مركزها اليها . ويتحرك مركزها بحركة الحامل له . حصل لمركز التدوير بتحركها اعني بانتقال

[١٥٩ و]

جملة الدائرة الصغيرة وبحركتها ايضا على مركز نفسها حركة مركبة من هاتين الحركتين يظنّ انها بسيطة مستوية عند مركز الدائرة التي هي اكثر خروجاً عن موضع البصر وهي المسمّاة بمعدل المسير .

واما مركز التدوير اعني نقطة المماسّة المذكورة فقد يخال انه محمول على الدائرة التي

٩ — واحد .
١٠ — الحركتين متساويتين
١١ — الدائرتين الاولتين .

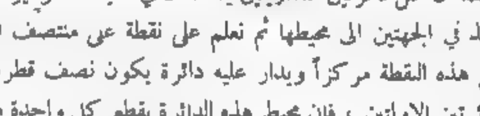
فلنقم على خط اب خطي[7] اج ، بد ويحيطان معه بالزاويتين الموصوفتين المتساويتين. ويوصل خط جد .

فاقول إنّ جد موازٍ لخط اب . برهانه أنّا نخرج خط اب على استقامة الى نقطة ه فإن كانت زاوية دبه الخارجة مساوية لزاوية جاب الداخلة على ما في الصورتين الاولتين فمن البيّن أنّ خطّي اج ، بد المتساويين يكونان متوازيين . فخطا اب ، جد كذلك واما إن كانت

[١٥٨ ظ]

الزاويتان المتساويتان هما الداخلتين اللتين في جهة واحدة اعني زاوية جاب مساوية لزاوية دبا كما في الصورتين الباقيتين فنخرج من نقطة د خطاً موازياً لخط اج وليلقى خط اب على نقطة ز .

فمن اجل ان اج موازٍ لدز تكون زاوية جاب مثل زاوية دزه . فلذلك تكون دزب مثل زاوية دبز . فخط دز مسوٍ لخط دب اعني اج وموازٍ له. فخطا اب ، جد متوازيان . وذلك ما اردنا بيانه .



ومن ذلك ايضا ان كل دائرتين متساويتين يتقاطعان في بسيط مستوٍ يوصل بين مركزيهما بخط مستقيم وينفذ في الجهتين الى محيطها ثم نعلم على نقطة على منتصف الخط الذي بين مركزيهما ونجعل هذه النقطة مركزاً ويدار عليه دائرة يكون نصف قطرها مساويا لنصف قطر احدى[8] الدائرتين الاولتين ، فان محيط هذه الدائرة يقطع كل واحدة من القطعتين اللتين تقعان من الخط المستقيم المار بالمراكز فيما بين محيطي الدائرتين بنقطتين نصفين .

٧ — خطا .
٨ — احد .

وليس لتحقيق ذلك طريق سوى الامتحان بالرصد في الوقت بعد الوقت . ولهذا يجب ان نختار من الارصاد ما يقرب منّا زمانه لكيلا يكون القـــدر الذي يفوتنا مضاعفا[1] مرات كثيرة .

ولما لم يكن لاهل زماننا وملوك عصرنا ومن له ابسيطة[2] رغبة في هذا العلم وقصر بنــا نحن ضعف الحال وكثرة العيال وقلة المساعد فلذلك لم نتكلم فيها من غير امتحان كما يفعل مصمو[3] الزيجات بان يزيدوا او ينقصوا من عند انفسهم بلا دليل ولا حجة سوى جهلهم بالطريق التي استخرجت بها هذه الامور . وانما حمَلَتهم[4] على ذلك كونهم يرون الخــلاف الواقع في كتب اهل هذه الصّناعة فاختار كل واحد منهم اوساطاً من نفسه فوضعها .

فبذلك صارت زيجاتهم على ما يرى من التناقض . ونعود الى كلامنا في افلاك الكواكب فنقول :

إنّ السبب الذي من اجله صار مركز التدوير يرى انه محمول على فلك خارج المركز ويرى مسيره المستوي عند مركز فلك آخر غير الذي هو محمول عليه ان نقطة مركز التدوير التي يظن بطلميوس انّها بسيطة ليست كذلك . وانما هي حركة مركبة من حركتين بسيطتين مستويتين على مركزين غير المركزين الموصوفين اعني مركز الحامل ومركز معدّل المسير اللذين ذكرهما .

لكن فلك التدوير اذا تحرك بالحركتين اللتين سنوضحهما فانه سيحصل من تركيبهما حركة مستوية تخال انها بسيطة عند مركز معدّل المسير . ونقدم لذلك تذكرةً نافعة فنقول :

إنّ كل خط مستقيم نقيم عليه خطين[5] مستقيمين متساويين[5] في جهة واحدة فيصيران زاويتين من الزاويا التي تحدث مع الخط امّا الداخلة مع الخارجة واما الداخلتان اللتان في جهة واحدة متساويتين[6] ثم يوصل بين طرفيهما[6] بخط مستقيم فانه يكون موازيا للخط الذي قاما عليه .

١ – صححت على الهامش
٢ – البسطه في المخطوط .
٣ – مصنفوا
٤ – كذا .
٥ – خطان مستقيمان متساويان
٦ – عبارة مكررة

f.160r

by Ptolemy for the distances between the deferent center and the equant from point *Q*, i.e. the center of the universe, for any planet, then what appears of these motions will be in agreement with what appeared to him (i.e. Ptolemy) by observation.

Appendix

[١٥٧ ظ] ىما الهيئة الصحيحة التي يتهيأ بها اصابة ما يخرج بالارصاد ويشاهد بالعيان ويجري على الاصول الموضوعة من غير مخالفة لشيء منها فنحن مثبتوها بابسط ما نقدر عليه. ونبين وضع الأكر التي تكون عنها الحركة البسيطة المتصلة على أن حركاتها مستوية عند مراكزها. والحركة المستوية هي التي يقطع المتحرك بها في الازمان المتساوية زوايا متساوية عند مركز المحرك له. والمختلفة هي التي ليست كذلك.

وينبغي ان تعلم أنّ إصابة مثل هذا الامر الجليل على الوجه الصواب في اعلى مراتب القوى الفكرية البشرية وهو تمام بالحقيقة للجزء النظري من التعاليم.

والذي ينبغي ان يسلّمه الباحث في هذا العلم هي الارصاد القديمة التي يظن بها الصحة مثل ارصاد ابرخس وبطلميوس اذ كان ممن يوثق بعلمهما وعملهما. فلنسلم ما اورداه من هذه الارصاد وهي التي عليها كان يعمل هو ايضا وعليها عمل حسابه الذي اخرجه بطريق [١٥٨ و] الخطوط والاوساط وهي المنتزعة من ازمان الادوار.

فاما الزمان الدوريّ ومقدار مسير كل كوكب في يوم يوم بالوسط والخاصة فإن تحقيقه موقوف على الامتحان فلا يصار اليه بغيره. واصابته بغاية التدقيق يعسر بل لا يمكن ان يدرك على الاستقصاء بحيث لا يفوت فيها ولا القدر اليسير. ومتى فات فيها مقدار ما وان قلّ فانه اذا مرّ عليه زمان طويل ظهر ظهوراً بيّناً ويزداد كلما طالت عليه المدة.

then its center will be point L and the sphere will be called the director *(al-mudir)* sphere of the epicycle.

Let this sphere be sunk into the thickness of (another) sphere whose curved parallel surfaces are around center K, so that it is tangent to its parallel surfaces in such a way that the surface of the director is tangent to its outer and inner surfaces. That sphere is called the carrier sphere (i.e. deferent).

When this sphere makes a full revolution the center of the director will then describe a circle whose center is point K, and that is the (above-)mentioned middle circle.

And as the director moves around center L, the epicycle of the planet, i.e. point E, will describe the small circle which is inside the sphere of the director, i.e. the (above-)mentioned circle ASE.

Now if the deferent moves uniformly, point L will move along the circumference of the third (circle) LNM whose center is point K. It will then move through its motion the sphere of the director. With the motion of the director sphere the center of the epicycle will also uniformly move along the small circle ASE and around its center, i.e. point L, at the same speed as point L.

So if point L moves along the circle LNM to point N and (then to) M on the left-hand side of circle LNM, then point E will move on circle ASE on the right-hand side to point O, then to point H.

Now if you imagine the situation as we described it, let the center of the director and the epicycle be at any assumed position. Then we join lines KFN, $DZOC$, and NOR to the circumference of the epicycle.

Then I say that the two lines KFN (and) $DZOC$ are parallel.

Its proof is that arc LN of circle LNM in all positions of point L, i.e. N of circle LNM, is similar to arc FO of the small circle. Then the two angles EKN and FNO are equal. And lines KN and DO are parallel. Then $ADO =$ angle LKN. And the motion of point E, i.e. O, around center D is similar to the motion of point L, i.e. N, around center K at any assumed time and place.

But the motion of point N around center K is uniform, hence the motion of point O around center D, i.e. the center of the equant, is uniform. This resulting motion of point O around center D is composed of the two uniform motions of points L and E, i.e. N and O.

That demonstrates what we said, that if point E moves with the sum of the two motions mentioned (above), it will have a uniform motion with respect to point D and equal in speed to the motion of circle LNM.

If the eye is assumed to be at point Q of line TG, and its distance from T were to be equal to the distance of point T from point D, then these distances, when their values are of the same quantities assumed (over a millenium before)

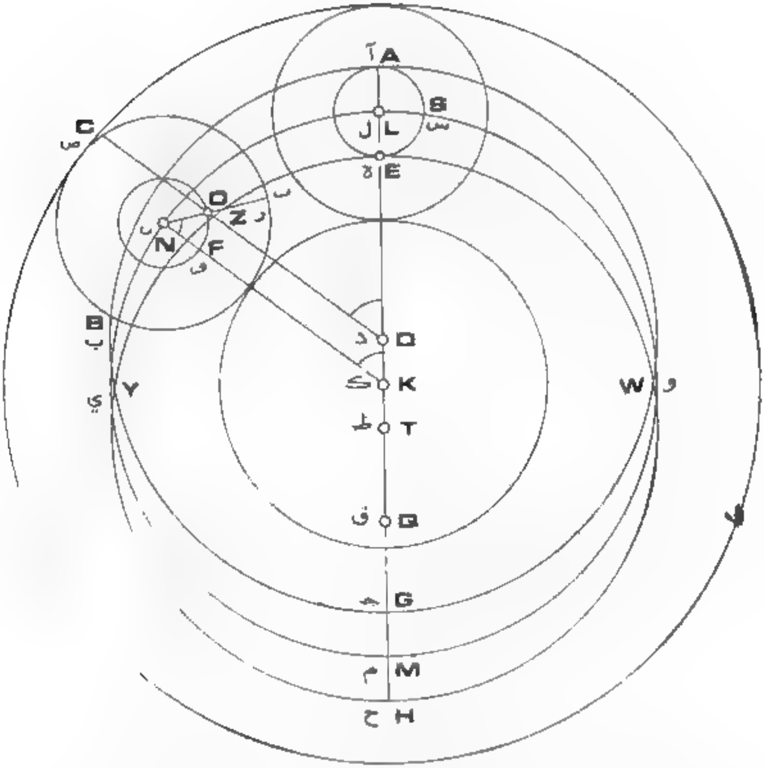


Figure 2. The planetary model of Shaykh Imām as reconstructed from Marsh 621.

by as much as the distance between the two centers, and if the center of the epicycle of the planet were imagined to be at the point where the small circle is externally tangent to the one of the two original circles whose center is closer to it, then if the small circle moves and with it the point of tangency, i.e. the center of the epicycle, in the direction opposite to that of the motion of the center. And if the center moves with the motion of its deferent, then the center of the epicycle moves with its motion, i.e. with the motion

f. 159r

of the small circle and its own motion on itself, in a motion composed of these two motions in such a way that it is thought to be simple and uniform at the center of the circle that is more eccentric from the eye, which is called the equant.

As for the center of the epicycle, i.e. the point of tangency mentioned above, it looks as though it were carried along the circle whose center is closer to the point of sight, on account of the fact that the center of the epicycle will be on this circle at its two distances, i.e. its farthest distance from the eye and its closest distance to it. And since it is very close to its circumference at the remaining portions of its distances (*dhurwa*), that has led Ptolemy to believe that the center of the epicycle is coincident with its circumference, and it describes it with its motion (Fig. 2).

Let us give an example to illustrate (that) very clearly. Let there be two equal circles intersecting in the same plane. The first of them, which is called the equant by Ptolemy, has points *ABG* on it and its center is point *D*. The second, which he calls the sphere carrying the center of the epicycle (i.e. deferent), is circle *EZH* with center *T*. Let the two (circles) intersect at points *W* and *Y*. We join the line *DT* that passes through the centers and produce it to the circumference on either side. Let it intersect circle *ABG* at the points *E* (and) *H*. We then bisect line *DT* at point *K* and with it as a center we draw a circle with a distance *DA*, i.e. the radius of the first circle, and (mark) in it points *L*, *N*, (and) *M*. It will bisect each of the two lines *AE* and *GH* at points *L* and *M*.

With point *L* as a center and with distance *AL* we draw circle *ASE*. It will be tangent to circle *ABG* internally at point *A* and tangent to circle *EZH* externally at point *E*. Let

f. 159v

point *S* be on the right-hand side of the small circle.

It is obvious then that the radius of this circle, i.e. *EL*, is equal to line *DK*, i.e. half the line connecting the centers of the first two circles *ABG* and *EZH*.

If we then assume that the first two circles *ABC* and *EZH* are fixed, and that the sphere surrounding the epicycle of the planet is tangent to the epicycle,

But if

158v

the two equal angles were the interior ones that are on the same side, i.e. angle $GAB = DBA$ as in the remaining two cases, then we produce from D a line parallel to AG and let it meet line AB at point Z.

Since AG is parallel to DZ then angle $GAB = DZE$. Therefore $DZB = DBZ$ and line $DZ = DB$, i.e. AG and is parallel to it.

Then the two lines AB and GD are parallel, and that is what we wanted to show.

In the same way, if two equal circles intersect on a plane surface and their centers are joined with a straight line that is produced in both directions to their circumference, and if we mark the midpoint of the line joining their centers and make it a center of a circle whose radius is equal to the radius of either of the two circles, then the circumference of this circle cuts the two segments of the straight line that is between the two circumferences of the two circles at their midpoints.

This circle intersects each of the two circles at two points other than the points of their original intersection.

If we make the point at which this circle cuts the two segments that are between the two circumferences a center and with it draw a small circle tangent to the two original circles, then the diameter of this circle is equal to the distance between the centers of the two original circles.

When the center of the small circle moves along the circumference of the third circle, which is the middle one of the three circles, until it reaches the diametrically opposite position on this line, then the small circle will also be tangent to the two circles to which it was tangent in the previous position, internally and externally, so that it will be externally tangent to the one to which it was internally so, and conversely with the other circle.

If one were to imagine the center of the epicycle of a planet to be carried on the circumference of this small circle, and (the circle) itself were assumed to be moving around its center in the direction of the zodiacal signs on the upper arc, i.e. the direction of the movement of the center, and in the reverse on the lower arc, and if the two motions were equal and the two original circles assumed to be fixed, and the eye (*baṣar*) were assumed to be on the line that passes through the centers and distant from the center of one of the two circles

ḥān) and is not obtainable otherwise. Its accurate determination is very difficult and rather cannot be achieved with high refinement (*istiqṣā'*) in a way that no slight inaccuracy is incorporated into it. And when any amount (of error) is incorporated into it, even if it be small, it will become quite apparent after the passage of time and will increase as the time increases.

The verification of that can only be achieved through testing by observations time after time. For that reason we must select the observations that are close to us in time so that the amount that we miss (i.e. the error) does not get multiplied several times.

And since our contemporaries and the kings of our times and those who have the authority have no bent toward this science, and we ourselves are lacking on account of our weakness and the expenses of our dependents and the lack of a helper, we did not say anything about it (i.e. observation) without testing as would the authors of *zījes* do when they add and subtract on their own without any evidence nor do they have any proof except their ignorance of the method by which these things are derived. They are (encouraged ?) to do so by what they see of the variations in the books of the people of this science and hence each of them selects mean motions for himself and sets them down.

For that reason the contradictions in these *zījes* are obvious. But let us return now to our discussion of the planets and say:

The center of the epicycle appears to be carried by an eccentric sphere, and its motion appears to be uniform with respect to the center of a sphere other than the one by which it is carried on account of the motion of the epicycle center which Ptolemy thinks is simple, but it is not so. (On the contrary) it is composed of two equal and uniform motions around two centers other than the ones described above, i.e. the centers of the carrier (deferent) and of the equant that he had mentioned.

But when the center of the epicycle moves with the two motions that we will describe the resulting uniform and composite motion will look as if it is simple with respect to the center of the equant.

Let us then introduce that with a useful reminder (*tadhkira*) by saying: Every straight line upon which we erect two equal straight lines on the same side so that they make two equal angles with the (first) line, be they alternate or interior, if their edges are connected, the resulting line will be parallel to the line upon which they were erected.

Erect on line AB the two lines AG and BD so that they surround with it the two equal angles described (above). Let line GD be connected.

Then I say: Line GD is parallel to line AB. Its proof is to produce AB to E. Then if the exterior angle DBE is equal to the interior angle GAB as in the first two cases, it is obvious that the two equal lines AG and BD are parallel.

"Some of the esteemed modern workers in this science (*ṣināʿa*) say in this place: If something is to be taken as a reference point for any motion, it must be stationary with respect to the moving thing so that motion will be due only to the moving body as it draws away from it or comes close to it."[5]

This same statement is made by Shaykh Imām in Marsh 621 in the relevant discussion of the moving center of the lunar deferent and which he uses as his own axiom to begin his new model. Furthermore, Quṭb al-Dīn, as usual, takes issue with this statement, hence proving that the author of Marsh 621 is a different person. In addition, this demonstrates that the work of Shaykh Imām was available to the Marāgha scholars and was actually incorporated into their works.

In what follows we give a translation of the text appended to this paper, from Marsh 621, fol. 157v-160r, attempting to be as literal as possible, only inserting a few explanatory words in brackets here and there to facilitate comprehension on the part of the reader.

Translation

f. 157v

As for the correct astronomy which agrees with what is obtained by observation and is apparent to the eye and (also) agrees with the accepted principles without any variation, we will explain it in the simplest way we can. We will also show the position of the spheres, which produce the continuous simple motion that is uniform with respect to their centers. The uniform motion is the one through which the moving (body) describes equal angles at the center of its mover in equal times. The non-uniform one is the one that is not so.

You must know that achieving such a momentous result in a correct fashion is of the highest human intellectual degrees and it is actual perfection of the theoretical part of the mathematical (sciences).

The researcher ought to accept in this science the ancient observations that he thinks are true, such as those of Hipparchus and Ptolemy, for they were trustworthy in knowledge and in practice. Let us accept what they have recorded by way of observations through which he (i.e. Ptolemy) himself used to work and upon which he based his computations, that he derived through

f. 158r

geometry (*khuṭūṭ*), and mean motions that are taken from periods of revolution.

As for the period of revolution and the daily motion of the planet in mean longitude (*wasaṭ*) and in anomaly, its verification depends upon testing (*imti-*

5. We transcribe here the text from Marsh 621, fol. 124v:1-3, to facilitate the comparison.

« إن الشيء الذي يفرض علامة لمبدأ حركة متحرك يجب ان يكون ساكناً بالنسبة الى المتحرك ليكون تبعد المتحرك عنه وتقربه اليه انما هو بحركة المتحرك وحده » .

Quṭb al-Dīn's text comes from the *Idrāk*, British Mus. Add 7482, fol. 52v 10-12.

this paper. We summarize here the tentative results reached so far and reported in the article mentioned above.[3]

The author of Marsh 621, at this stage, can be called al-Shaykh al-Imām as the scribe refers to him on fol. 126r. He must have lived between 1138 A.D. and 1272 A.D.

Shaykh Imām did not participate in the activities of the Marāgha observatory, for he says that he has no access to new observations. Hence he was probably writing before 1259. This author suspects that Shaykh Imām was not Mu'ayyad al-Dīn al-ʿUrḍī, a likely candidate.*

Shaykh Imām was not known to Ibn al-Shāṭir except through the works of Quṭb al-Dīn al-Shīrāzī.

And finally, it is highly probable that the "Ṭūsī couple" grew out of Imām's model as a logical consequence.

Due to the historical significance of this source, this author has undertaken a full transcription of it, but will give here only the relevant section on the planetary model with an English translation for the benefit of the reader who is not familiar with Arabic.

Quṭb al-Dīn and Shaykh Imām

The first reading of Marsh 621 revealed the identity of Shīrāzī's planetary model and that of Shaykh Imām. A first working hypothesis, however, was to assume that Marsh 621 was some earlier work of Shīrāzī reproduced in the *Nihāyat al-'idrāk* of Quṭb al-Dīn in a different format. That hypothesis ran into immediate problems, for the author of Marsh 621 is referred to as deceased by 1272 A.D., as was already noticed by Goldstein and Swerdlow,[4] whereas Quṭb al-Dīn was still writing in 1281 A.D. and lived till 1311 A.D.

The task remained, however, to prove beyond doubt that the phrase *qaddasa 'Allāhu rūḥahu* (May God bless his soul) is to be taken literally, and hence to establish Shaykh Imām as different from and earlier than Quṭb al-Dīn.

Hence it was necessary to examine the work of Quṭb al-Dīn with this question in mind. The present writer did so, braving Quṭb al-Dīn's "exasperating traits" of prolixity and repetition, coming upon the following passage of the *Nihāyat al-'idrāk*:

« قال بعض افاضل المتأخرين من اهل الصناعة ههنا ان الشيء الذي يجعل علامة لمبدأ حركة يجب ان يكون ساكناً بالنسبة الى المتحرك ليكون تباعد المتحرك عنه وتقاربه اليه بحركة المتحرك وحده » .

3. These results were first reported on December 12, 1978, in a commentary read at the Boston Colloquium for the Philosophy of Science. The full text of the commentary will be published in the proceedings of the Colloquium.

4. *Op. cit.*, p. 146.

* *Note added in proof*: In an article appearing in *Isis* the present author has now established that al-Shaykh al-Imām was indeed al-ʿUrḍī (d. 1266) and that the text preserved in Marsh 621 was written before the building of the Marāgha observatory in 1259.

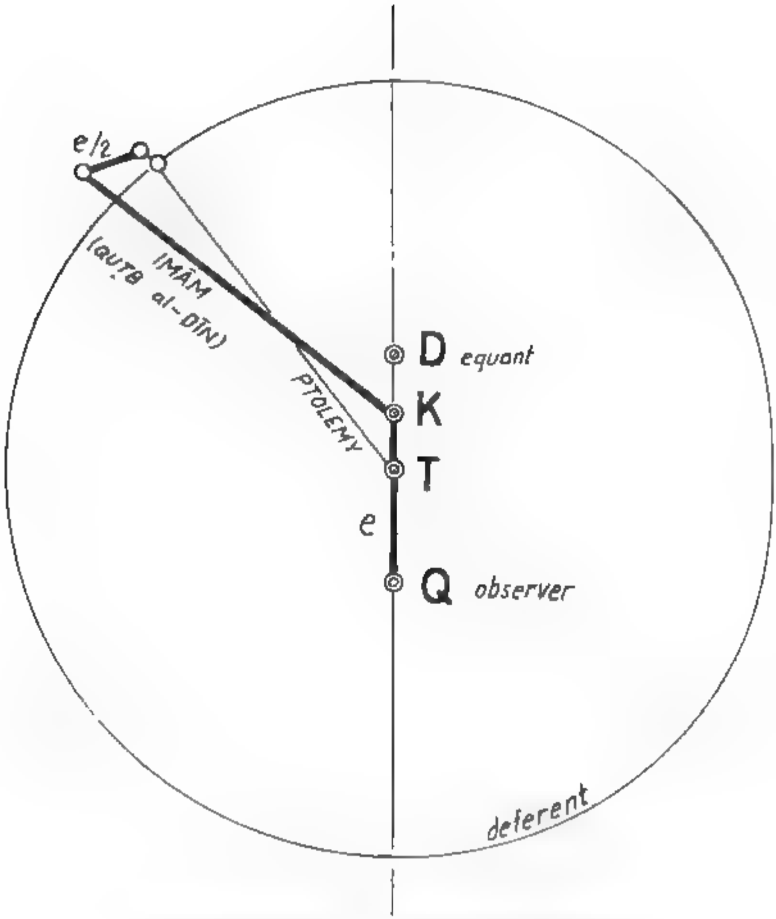


Figure 1. Sketch (not to scale) illustrating the two models.

The Original Source of Quṭb al-Dīn al-Shīrāzī's Planetary Model

George Saliba*

Introduction

A STUDY[1] published some twelve years ago reviewed the information then available concerning late medieval planetary theory. In this article, more space was devoted to the work of Quṭb al-Dīn al-Shīrāzī (fl. 1280 A.D.) than to any other individual. The model he uses for all the planets except Mercury differs from those of his contemporaries, Naṣīr al-Dīn al-Ṭūsī and Ibn al-Shāṭir. It was then remarked that perhaps the unique feature of Quṭb al-Dīn's arrangement had not been invented by him, but had been inherited from a predecessor.

This paper introduces a text,[2] anterior to that of Quṭb al-Dīn, in which the distinctive device is fully described and motivated. As such, it constitutes the earliest successful effort thus far discovered to eliminate a supposed fault in the Ptolemaic system. It was a belief widely held in antiquity that the motion of any celestial body must be circular and uniform, or a combination of uniform circular motions. Ptolemy's equant device (see Fig. 1 below), although imposed by the facts of observation, violated this principle. The mechanism here explained conforms fully to the requirement of uniform circularity, retains the effect of the equant and yields predictions differing only slightly from those obtainable with the Ptolemaic model.

In a separate article, the involved problem of authorship and priority as well as the relationships among the members of the "Marāgha School" has been treated in some detail, and further research is still going on to unravel the intricate relationships and historical questions involved. Nevertheless, there seems to be no way in which future research can change the thesis of

*Department of Near Eastern Languages and Literature, Faculty of Arts and Sciences, N. Y. U., Washington Square, New York City 10003.

1. E. S. Kennedy, "Late Medieval Planetary Theory", *Isis*, 57 (1966), 365-378.

2. Bernard R. Goldstein and Noel Swerdlow, "Planetary Distances and Sizes in an Anonymous Arabic Treatise Preserved in Bodleian Ms. Marsh 621", *Centaurus*, 15 (1970), 135-170. The author wishes to thank Prof. N. Swerdlow of the University of Chicago for bringing this Ms to his attention. The author is also indebted to the courtesy of Prof. B. Goldstein of the University of Pittsburgh for allowing him to investigate this manuscript.

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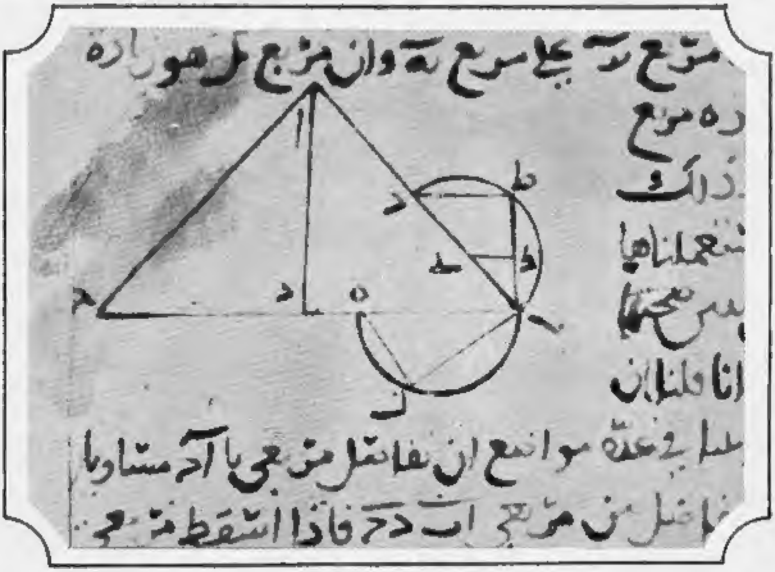

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